

# Ensemble-based Assimilation of HF-Radar Surface Currents in a West Florida Shelf ROMS Nested into HYCOM and filtering of spurious surface gravity waves.

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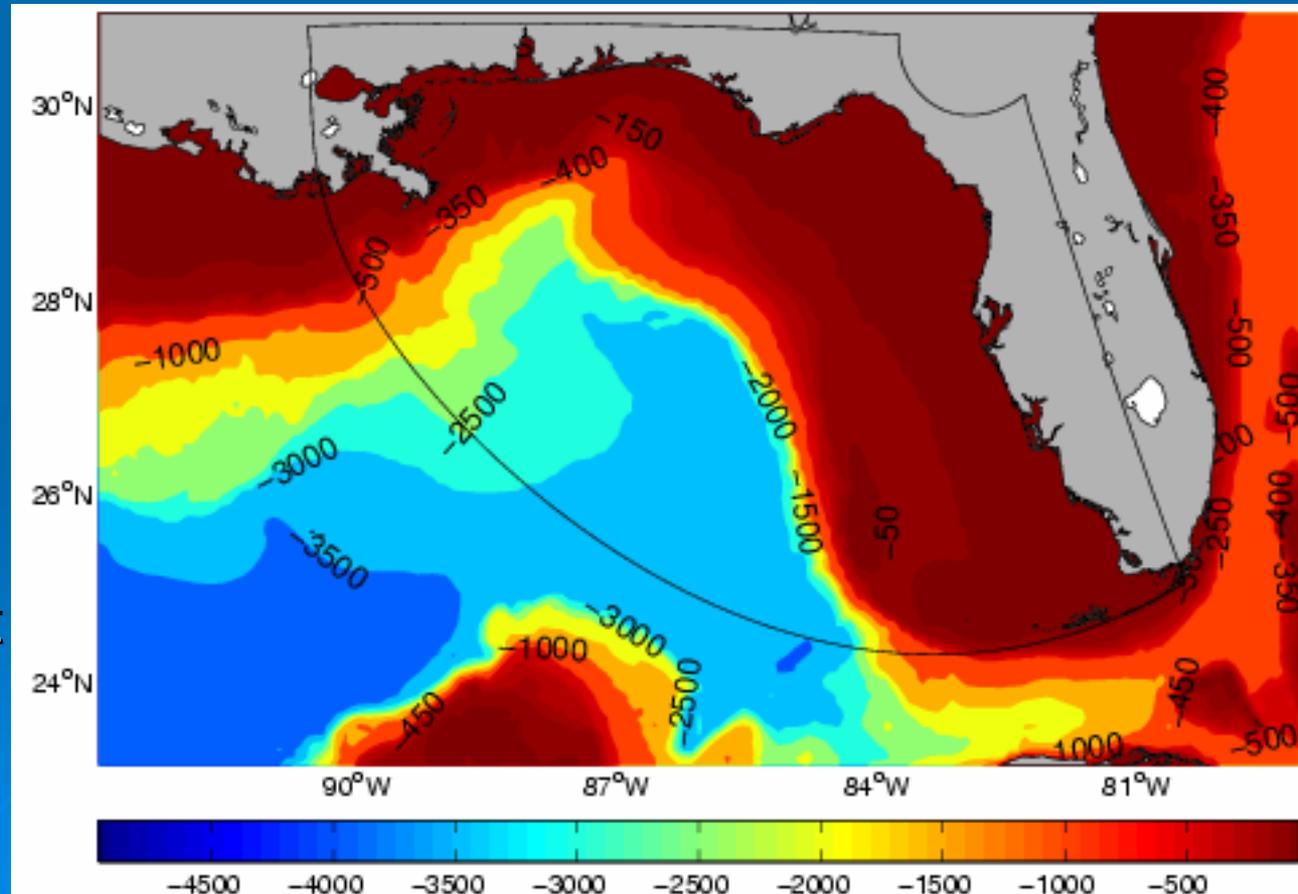
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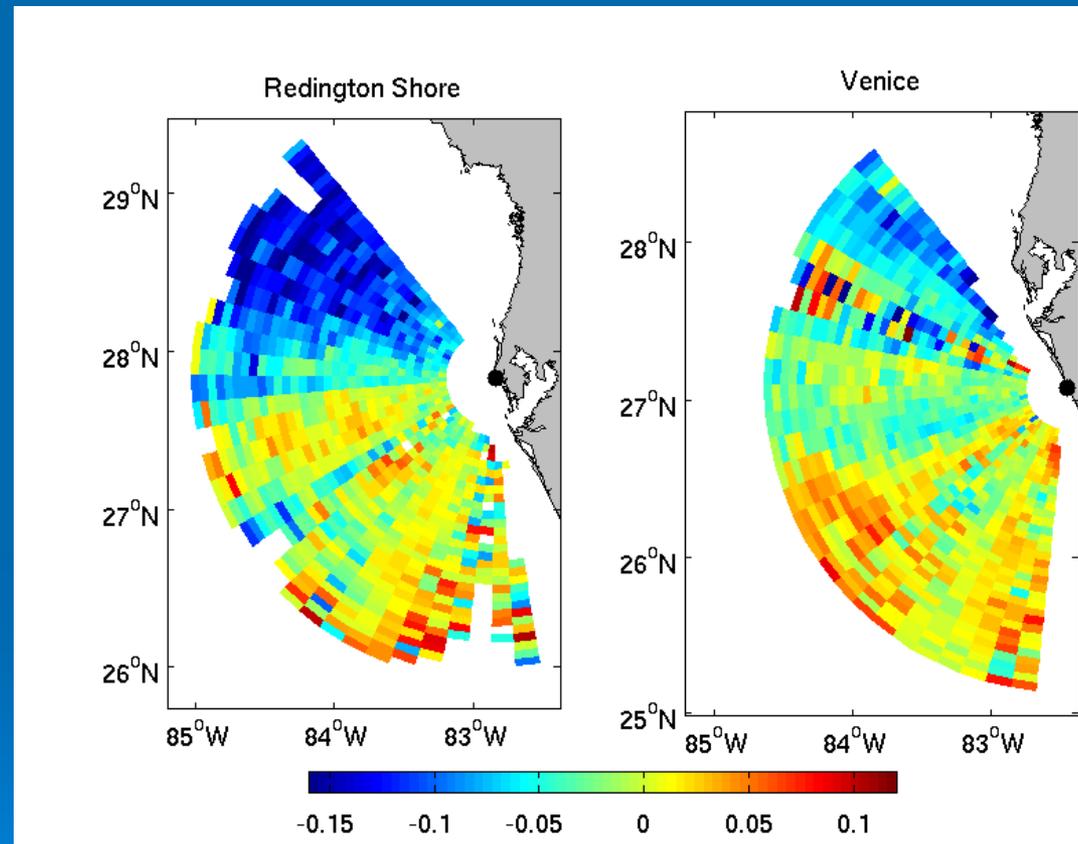
# WFS domain

- Black line shows the boundary of the model domain
- Domain is composed by:
  - Broad shelf
  - Deep ocean part
- Both regions are separated by a steep shelf break
- The Loop Current is the dominant large-scale feature



# Data assimilation: Observations

- HF-Radar radial surface currents maps
- detided
- 2-day averaged, but still “noisy”
- error estimate provided by instrument

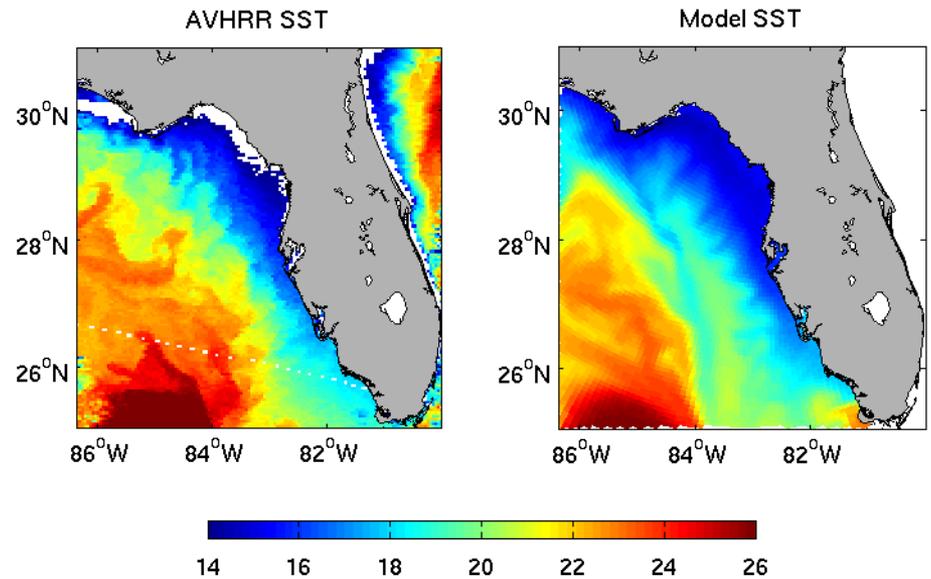
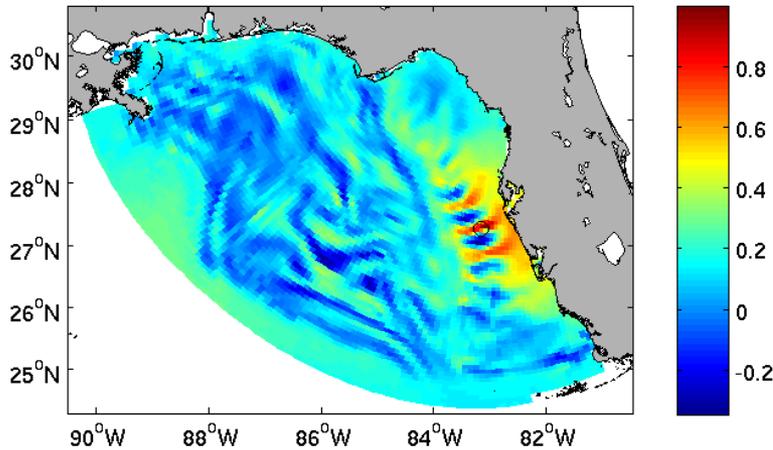


Radial velocities measured from the Redington and Venice sites on December 9, 2005. Positive values represent current towards the antenna.

# Model error covariance

- 100-member ensemble of wind fields
  - EOF analysis of the u and v wind components
  - random perturbations proportional to spatial EOFs
- For each wind field, the WFS ROMS model was integrated for 30 days
- The resulting ensemble was used for the assimilation of HF Radar currents
- Error covariance assumed constant in time -> “OI-approximation”.

# Ensemble covariance



Correlation between the u-velocity at a specific location marked by the circle and the u-velocity at all other model grid points.

AVHRR SST and model SST on January 29, 2004

The velocity error covariance on the shelf is closely related to the presence of the meandering front on the shelf. The covariance structure is a superposition of various ensemble members with different phase.

# SEEK analysis

Analysis:  $\mathbf{x}^a = \mathbf{x}^f + \mathbf{K} (\mathbf{y}^o - \mathbf{H}\mathbf{x}^f)$

Kalman gain:  $\mathbf{K} = \mathbf{P}^f \mathbf{H}^T (\mathbf{H}\mathbf{P}^f \mathbf{H}^T + \mathbf{R})^{-1}$

For a reduced rank-error covariance:

$$\mathbf{P}^f = \mathbf{S}^f \mathbf{S}^{fT}$$

Eigenvalue decomposition:

$$(\mathbf{H}\mathbf{S}^f)^T \mathbf{R}^{-1} (\mathbf{H}\mathbf{S}^f) = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^T$$

Kalman gain can be written as:

$$\mathbf{K} = \mathbf{S}^f \mathbf{U} (\mathbf{I} + \mathbf{\Lambda})^{-1} \mathbf{U}^T (\mathbf{H}\mathbf{S}^f)^T \mathbf{R}^{-1}$$

# State vector

- The state vector includes:
  - elevation
  - horizontal velocity
  - temperature and salinity
  - 2-day averaged wind stress
- All variables are at the model native grid (curvilinear, Arakawa C).
- Why wind stress and not wind speed?

$$\left[ \nu \frac{\partial \mathbf{u}}{\partial z} \right]_{z=0} = \tau_s = C_d \|\mathbf{u}_a\| \mathbf{u}_a$$

# Filtering barotropic waves

- Filtering the analysis correction:

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{FK} \left( \mathbf{y}^o - \mathbf{Hx}^f \right)$$

- Shallow water equations:

- Three solutions:

- 2 inertial gravity waves
- Geostrophic equilibrium

$$\begin{aligned} \frac{\partial \eta}{\partial t} &= -\frac{\partial U}{\partial x} - \frac{\partial V}{\partial y} \\ \frac{\partial U}{\partial t} &= fV - gh \frac{\partial \eta}{\partial x} \\ \frac{\partial V}{\partial t} &= -fU - gh \frac{\partial \eta}{\partial y} \end{aligned}$$

# Filtering barotropic waves

- Amplitudes of the inertia-gravity waves are set explicitly to zero
- The filtered elevation is in geostrophic equilibrium satisfies:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{f^2}{gh} \right) \eta' = \frac{1}{h} \frac{\partial V}{\partial x} - \frac{1}{h} \frac{\partial U}{\partial y} - \frac{f^2}{gh} \eta$$

- Conserves potential vorticity locally
- Corresponds to the geostrophic adjustment solution after infinite time

# Uneven bottom topography

- Using the potential vorticity argument, the method can be extended to arbitrary topography
- However, the barotropic flow may cross isobaths -> generation of waves
- Variational approach using the topography as weak constrain:

$$E = \int_S \left[ \left( \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} - \frac{f^2}{gh} \eta - q \right)^2 + \sigma (J(h, \eta))^2 \right] dS$$

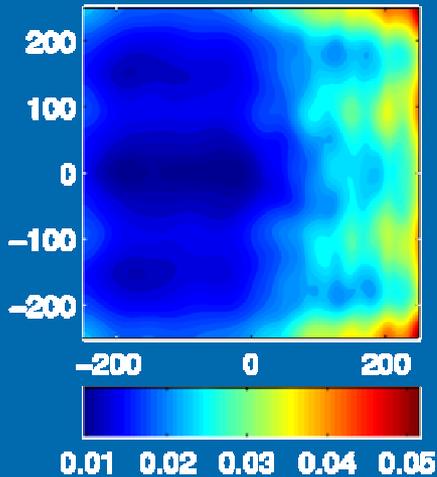
# Comparison with Incremental Analysis Update (IAU)

- Idealized coastal model with a shelf break
- Background flow along the shelf break in geostrophic equilibrium
- We add a random elevation perturbation to the model

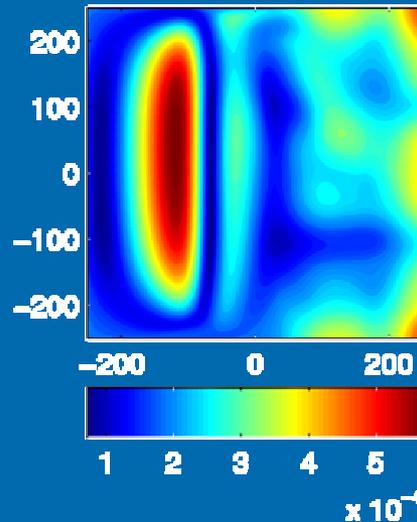


# Numerical experiments

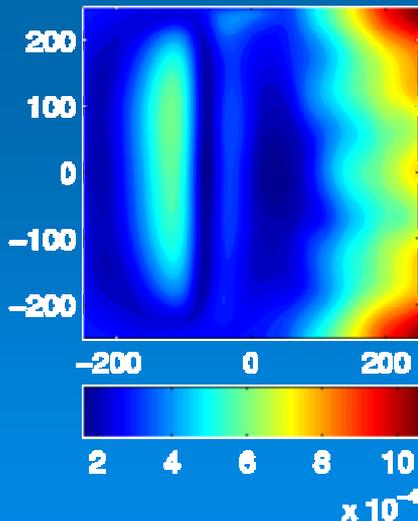
(a) Std. of elevation (original IC)



(b) Std. of elevation (variational filter)



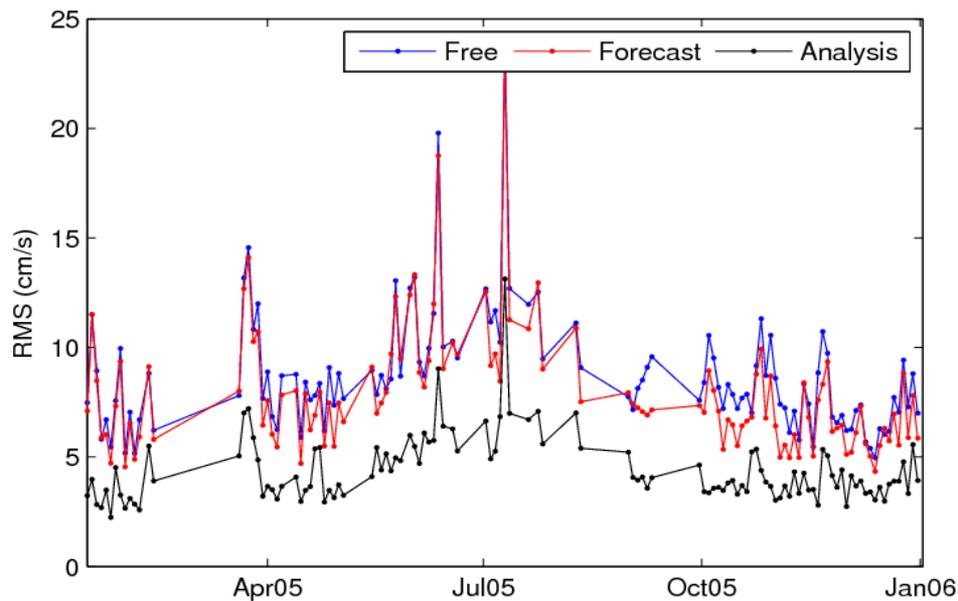
(c) Std. of elevation (IAU)



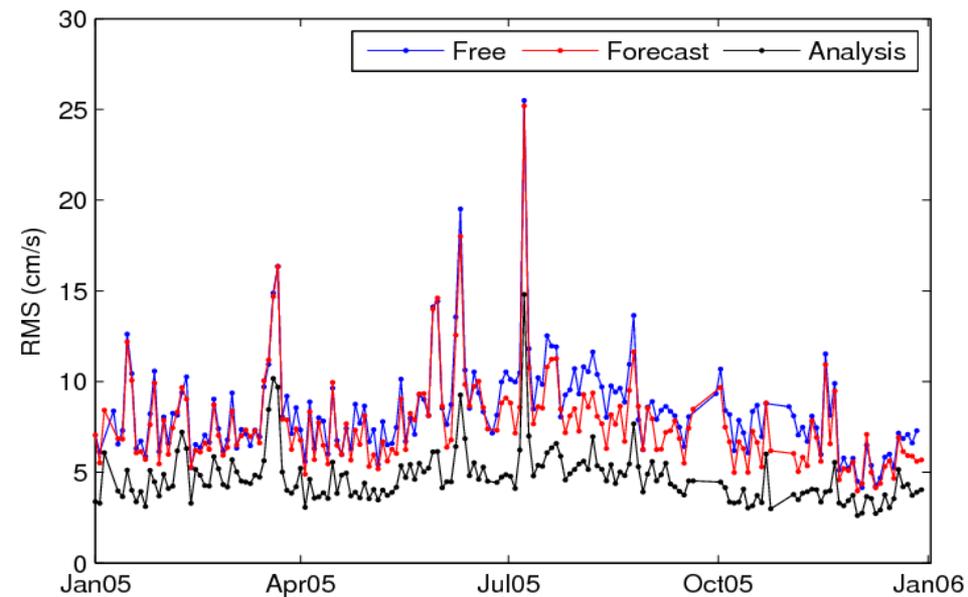
- Standard deviation of elevation integrated over time with different initial conditions
- The variational filter and IAU reduce substantially the elevation variation
- IAU works better in the open ocean while the variational filter is better on the shelf
- In average, the variance of the variational filter is lower than the IAU

# RMS error relative to the HF Radar currents

Redington



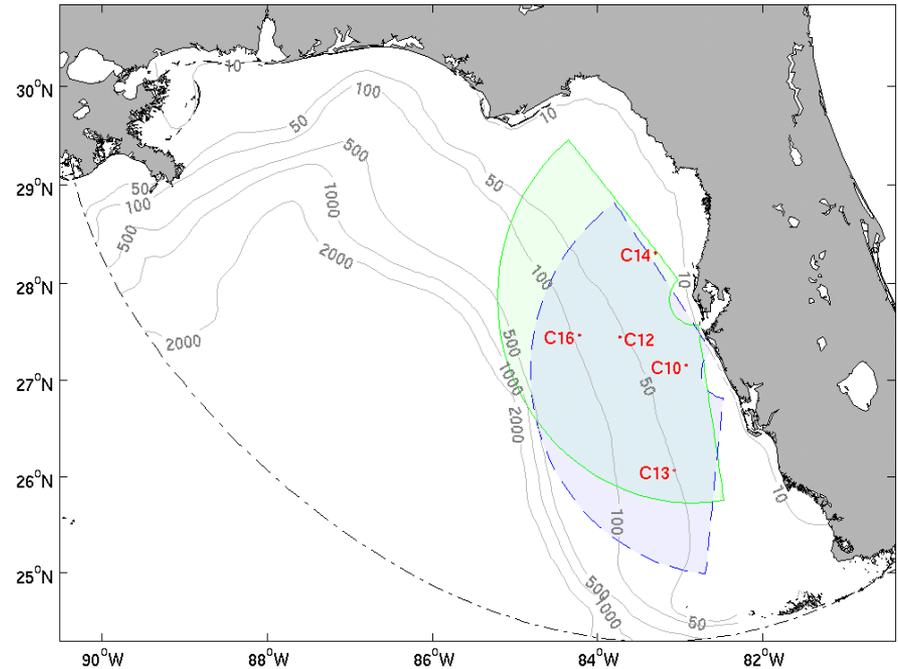
Venice



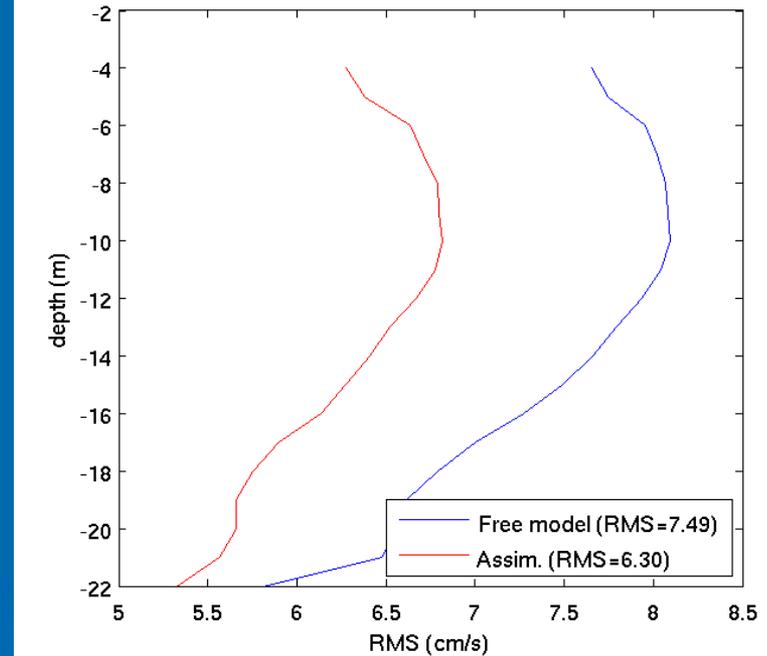
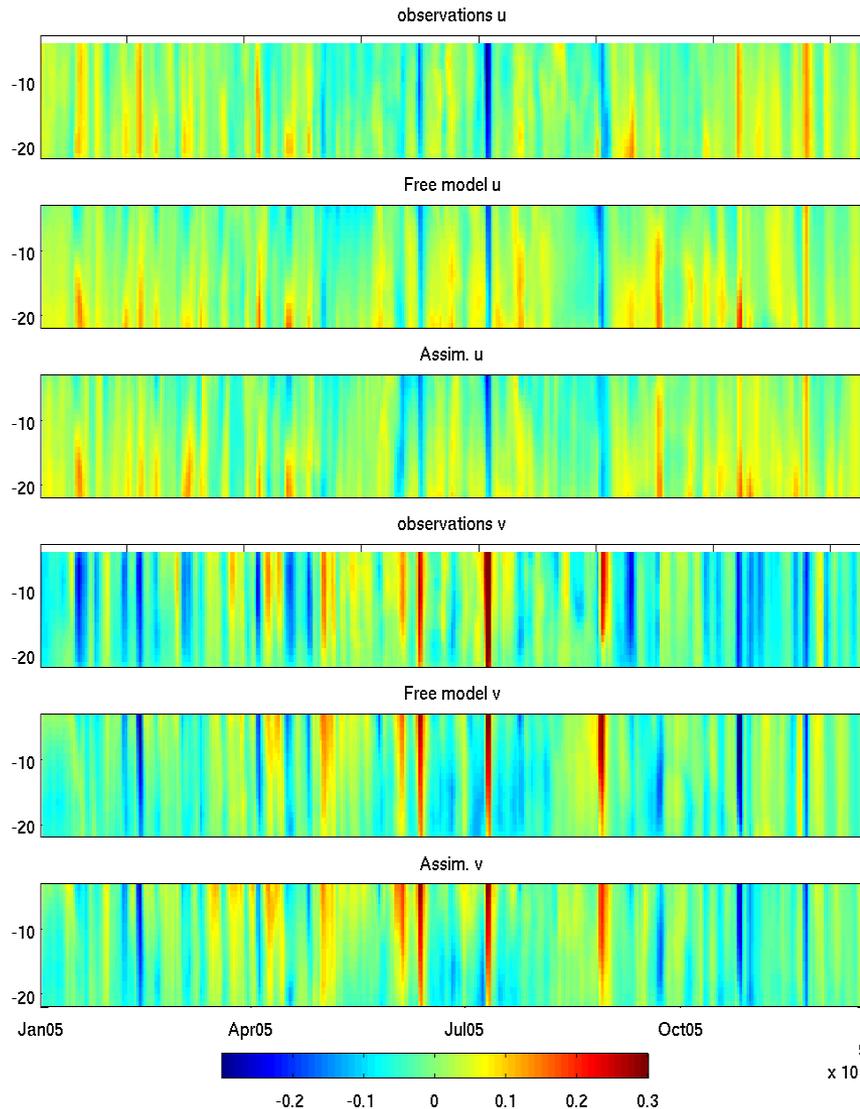
The RMS time series for the model run without assimilation (free model), the model forecast (before assimilation of CODAR data) and the model analysis (after assimilation) are shown.

# Comparison with independent observations

- Several ADCP sites on WFS shelf
- Error reduction at the surface is expected, but
- how does the error behave at depth?

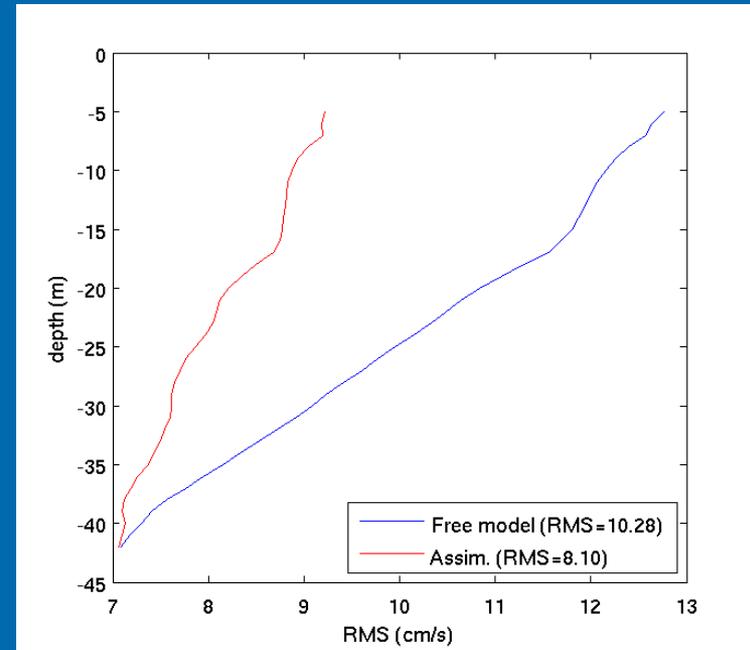
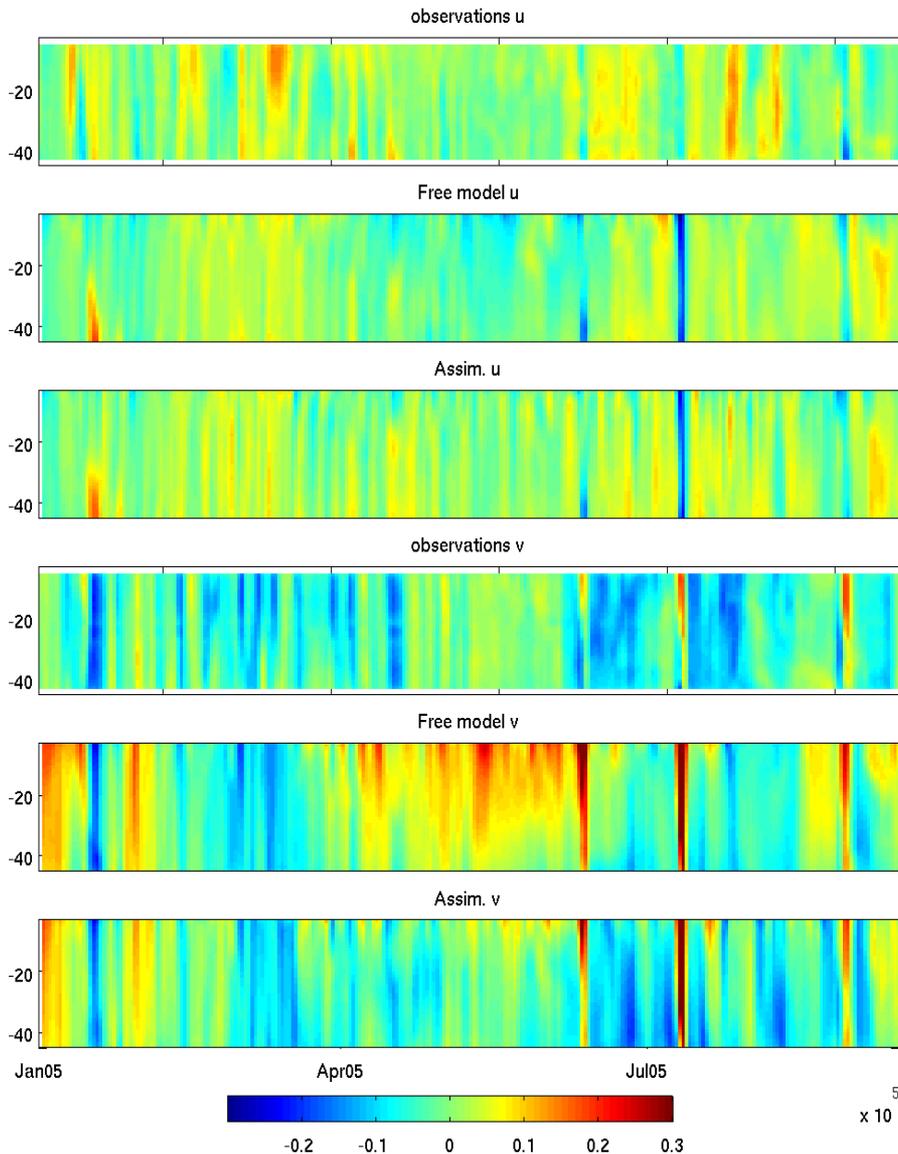


# ADCP observations from C10



- Free model already very close to observations.
- Time averaged RMS shows however that error is reduced.

# ADCP observations from C12



- Free model shows an unrealistic Northwestward current during summer which is corrected through the assimilation
- Except at the bottom, the time averaged RMS error is reduced.

# OCG Model products



## West Florida Shelf ROMS model

Bulletin (YYYY-MM-DD):

Variables:

Vector field:

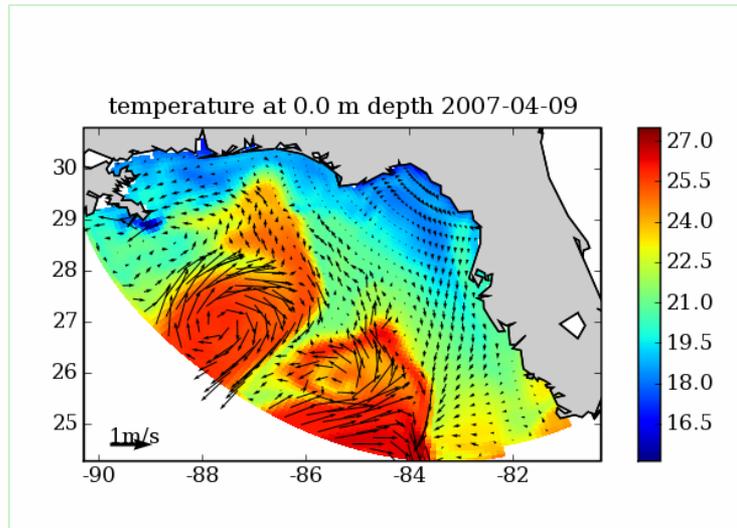
Depth: (m)

Forecast:

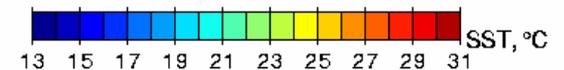
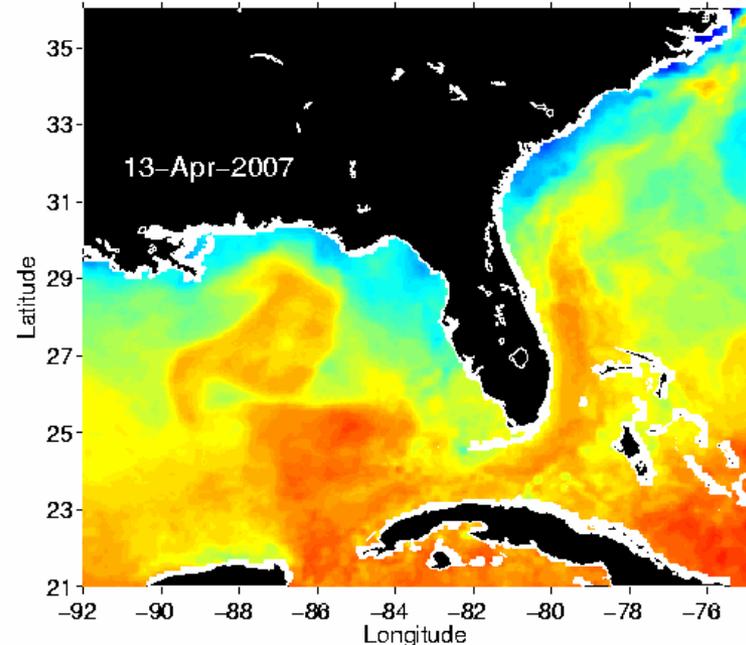
Squares can be dragged.

Longitude:

Latitude:



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# Compare the West Florida Shelf ROMS model with BSOP

Unit:

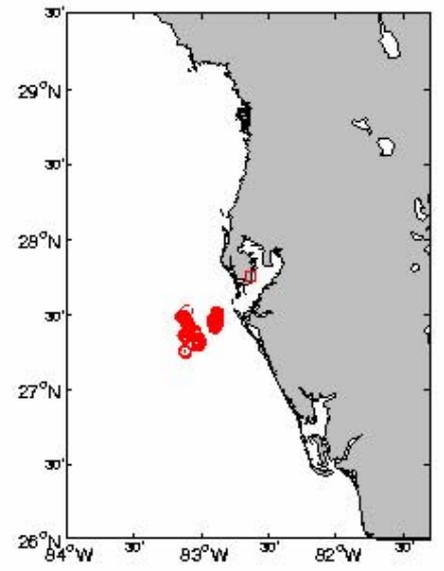
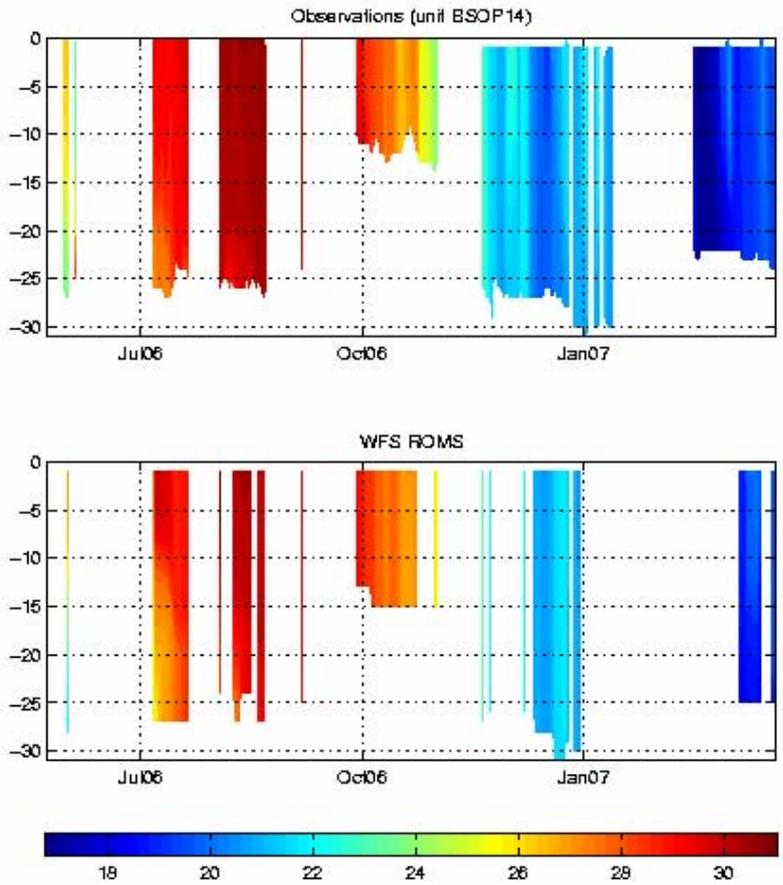
Start time:

End time:

Variable:

Color axis:  
 automatic  
 manual. Limits:

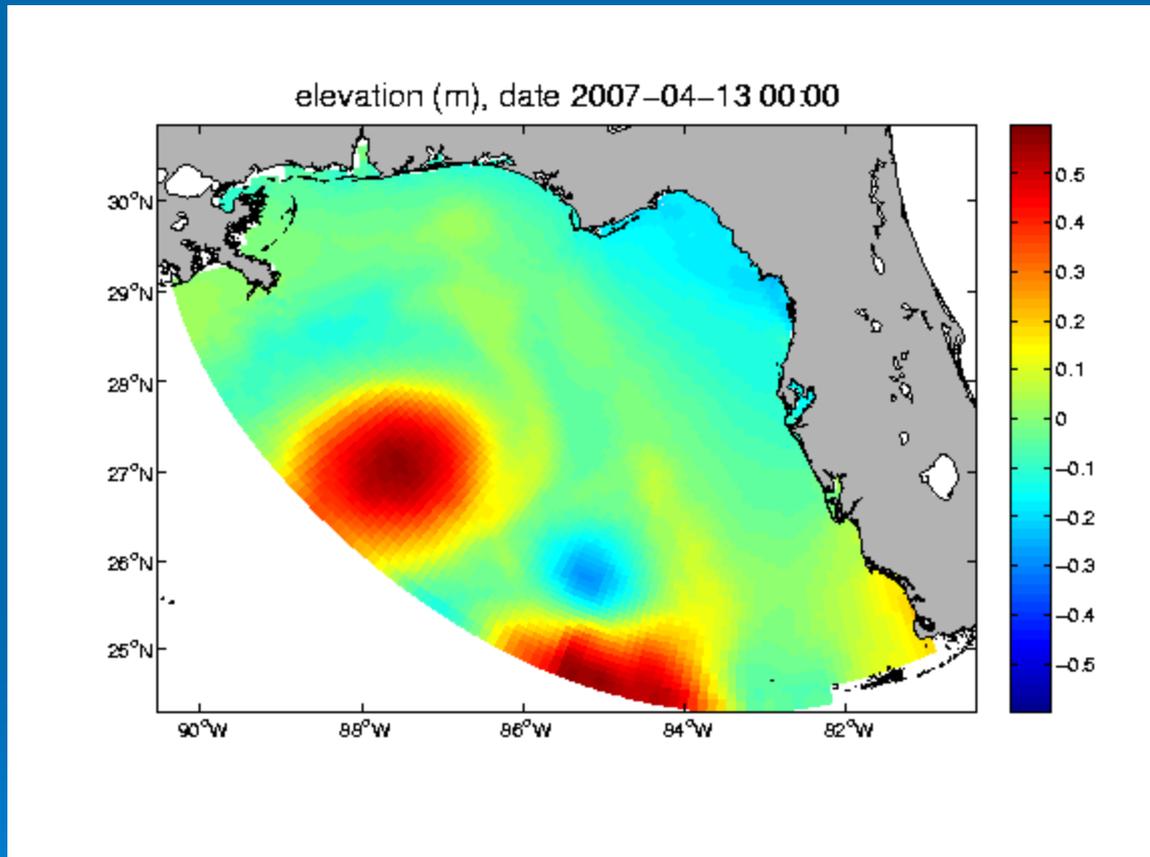
Finished.



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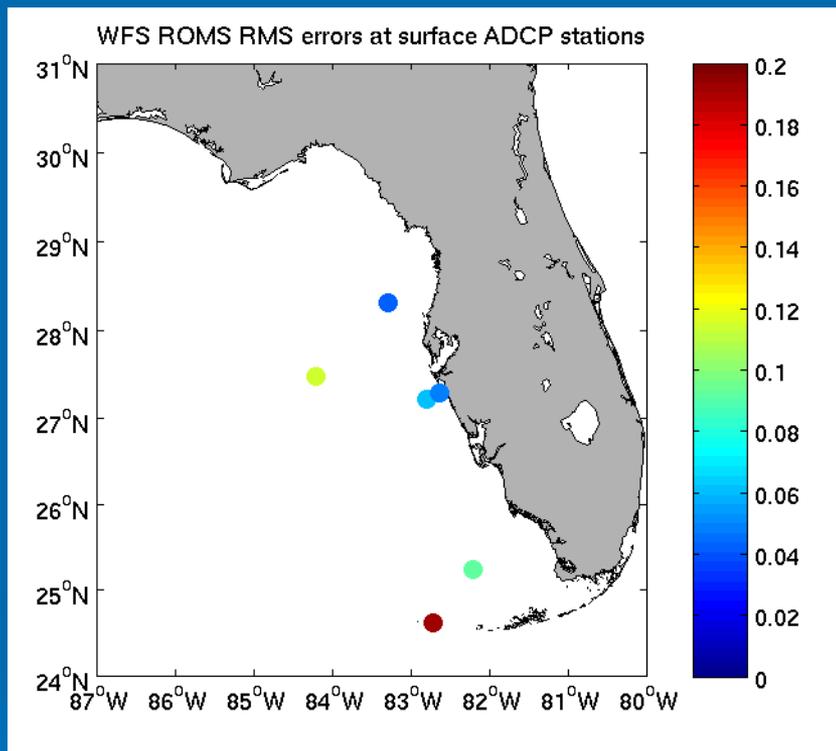
# Preliminary tidal model forecast



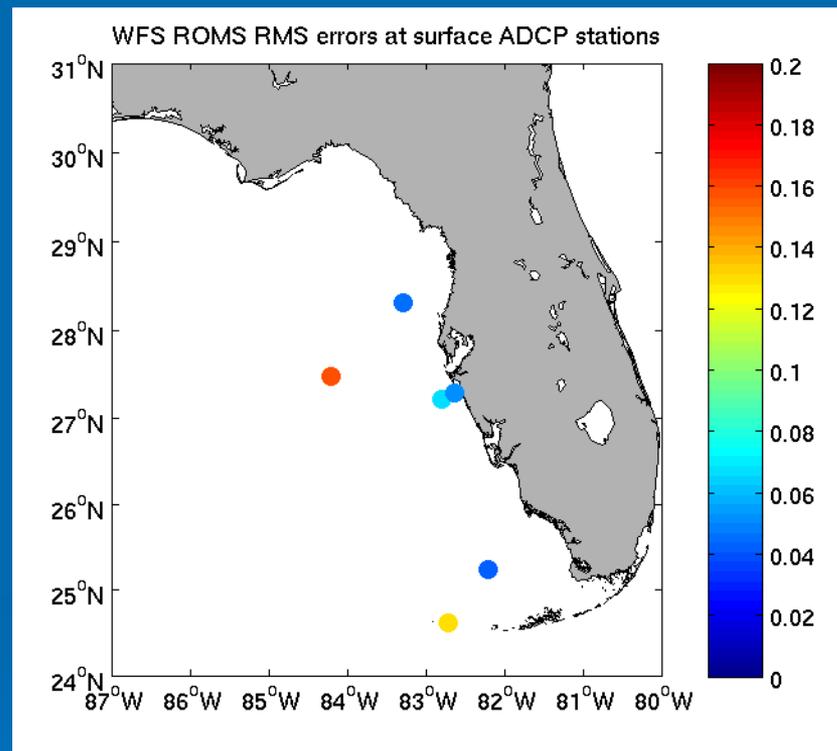
- Add tides to HYCOM elevation and velocity boundary conditions
- Model forecast in parallel to model without tides

# Sensitivity to Boundary Conditions

Atlantic HYCOM



GoM NCODA HYCOM



- Lowest RMS error on the shelf in all simulations
- With GOM NCODA HYCOM boundary improvements in C19 and C17 but degradation in C16
- Nested model bathymetry needs to be adapted to the outer model bathymetry (which is different in both HYCOM simulations) and involves certain parameters which can be optimized
- A more clear-cut comparison may be obtained

# Conclusions

- HF Radar currents is a promising dataset to constrain the circulation of coastal models.
- The ensemble-based error covariance contains rich small-scale structures near fronts which can be described as a superposition of ensemble members with different phase.
- The present study shows how forcing functions like the wind stress can be improved using a sequential assimilation scheme.
- A method for reducing barotropic waves introduced by DA is proposed.
- The proposed CODAR assimilation scheme is able to improve:
  - The 2-day velocity forecast
  - The velocity at depth
- Automated model verification with BSOP data is currently developed
- Experimental model runs with tides are implemented
- Atlantic HYCOM and GoM NCODA HYCOM simulations provide both adequate boundary conditions for the WFS.

# Sequential algorithm

- Data is assimilated every 2 days
- Model is started at  $t-2$  and run for 3 days
- Currents are averaged over  $t-1$  and  $t+1$
- Wind stress is also averaged over  $t-1$  and  $t+1$
- Analysis increment is computed based on the model error covariance expressed as an ensemble
- This correction is added to the instantaneous model field at  $t$  to produce a new initial condition (IC)
- The wind stress correction is applied uniformly to the wind forcing between  $t$  and  $t+1$

