Combining HYCOM, AXBTs and Polynomial Chaos Methods to Estimate Wind Drag Parameters during Typhoon Fanapi

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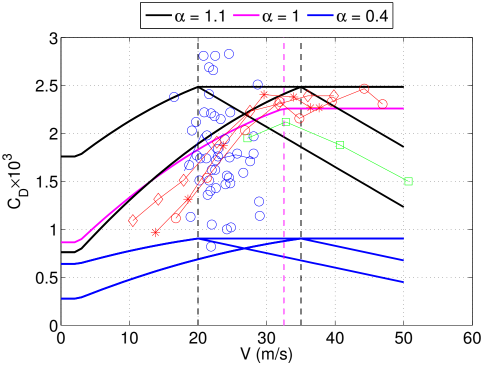
Outline



$$\begin{aligned}
\vec{\tau} &= \rho_a C_D V \vec{V} \\
C_D &= C_{D0} + C_{D1}(T_s - T_a) \\
C_{D0} &= a_0 + a_1 \tilde{V} + a_2 \tilde{V}^2 \\
C_{D1} &= b_0 + b_1 \tilde{V} + b_2 \tilde{V}^2 \\
\tilde{V} &= \max[\, V_{\min},\ \min(V_{\max}, V)]
\end{aligned}$$

C_D is drag coefficient
V is wind speed at 10 m.
C_D saturates for $V > V_{\max}$

- Blue circles: aircraft observations (French et al., 2007),
- red: wind tunnel (Donelan et al., 2004),
- green: drop sondes (Powell et al., 2003),
- magenta: HYCOM fit to COARE 2.5,
- Problem: $V_{\max}$ and $C_D^{\max}$ are not well-known and does C_D decrease for $V > V_{\max}$ as drop sondes suggest?

Inverse Modeling Problem

- Perturb C_D by introducing 3 control variables $(\alpha, V_{\max}, m)$

$$C_D{}' = \alpha C_D \text{ for } V < V_{\max} \quad (1)$$

$$C_D{}' = \alpha[C_D + m(V - V_{\max})] \text{ for } V > V_{\max} \quad (2)$$

- multiplicative factor $0.4 \leq \alpha \leq 1.1$
- vary $V_{\max}$ between 20 and 35 m/s
- m is a linear slope modeling decrease for $V > V_{\max}$ with $-3.8 \times 10^{-5} \leq m \leq 0$
- Use ITOP data to learn about likely distribution of α, $V_{\max}$ and m.

Bayes Theorem: $p(\boldsymbol{\theta} \mid \boldsymbol{T}) \propto p(\boldsymbol{T} \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta})$

- Likelihood: $\epsilon = \boldsymbol{T} - \boldsymbol{M}$ is normally distributed

$$p(\boldsymbol{T} \mid \boldsymbol{\theta}) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(T_i - M_i)^2}{2\sigma^2}\right) \tag{3}$$

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- σ^2 unknown, treated as hyper-parameter. Assume a Jeffreys prior

$$p(\sigma^2) = \begin{cases} \frac{1}{\sigma^2} & \text{for } \sigma^2 > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

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- Uninformed priors for α, $V_{\max}$ and m:

$$p(\{\alpha, V_{\max}, m\}) = \begin{cases} \frac{1}{b_i - a_i} & \text{for } a_i \leq \{\alpha, V_{\max}, m\} \leq b_i, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

where $[a_i, b_i]$ denote the parameter ranges.

Final Form of Bayes theorem

$$p(\{\alpha, V_{\max}, m\}, \sigma^2 | \boldsymbol{T}) \quad \propto \quad \left[\prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(T_i - M_i)^2}{2\sigma^2} \right) \right] p(\sigma^2)\, p(\alpha)\, p(V_{\max})\, p(m)$$

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- Build full posterior with Markov Chain Monte Carlo (MCMC) MCMC requires $O(10^5)$ estimates of M_i: prohibitive
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- Rely on Polynomial Chaos expansions to replace HYCOM by a polynomial series that could be either summed for MCMC or differentiated for the gradients.

What is Polynomial Chaos

- Series Representation of Model Output

$$M(\boldsymbol{x}, t, \boldsymbol{\theta}) = \sum_{k=0}^{P} M_k(\boldsymbol{x}, t)\, \psi_k(\boldsymbol{\theta}) \tag{6}$$

  - $M(\boldsymbol{x}, t, \boldsymbol{\theta})$: a model output (aka observable)
  - $M_k(\boldsymbol{x}, t)$: series coefficients
  - $\psi_k(\boldsymbol{\theta})$: orthogonal basis functions w.r.t. $p(\boldsymbol{\theta})$
  - mean: $E\left[M\right] = \langle M, \psi_0 \rangle = \sum_{k=0}^{P} M_k(\boldsymbol{x}, t) \langle \psi_k, \psi_0 \rangle = M_0(\boldsymbol{x}, t)$
  - Variance: $E\left[(M - E\left[M\right])^2\right] = \sum_{k=1}^{P} M_k^2(\boldsymbol{x}, t)$
- Basic Questions
  - How to choose ψ_k? Legendre polynomials
  - How to determine the coefficients M_k? Projection
  - Where to truncate the series, P ? Monitor Variance

How do we determine PC coefficients

- Series: $M(\boldsymbol{x}, t, \boldsymbol{\theta}) = \sum_{k=0}^{P} M_k(\boldsymbol{x}, t)\psi_k(\boldsymbol{\theta})$
- Projection:

$$M_k(\boldsymbol{x}, t) = \langle M, \psi_k \rangle = \int M(\boldsymbol{x}, t, \boldsymbol{\theta})\psi_k(\boldsymbol{\theta})\rho(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}$$

- Approximate integral with numerical Quadrature

$$M_k(\boldsymbol{x}, t) \approx \sum_{q=1}^{Q} M(\boldsymbol{x}, t, \boldsymbol{\theta}_q)\psi_k(\boldsymbol{\theta}_q)\omega_q$$

- $\boldsymbol{\theta}_q/\omega_q$ quadrature points/weights
- Quadrature requires an ensemble run at $\boldsymbol{\theta}_q$
- Here we Used Adaptive quadrature requiring 6-iteration levels for a total of 67 realizations

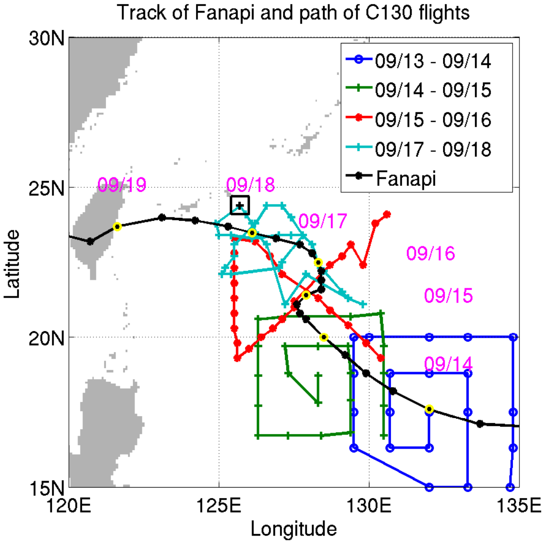


Figure: Fanapi's JTWC track (black curve) and paths of C-130 flights. The yellow circles on the track represent the typhoon center at 00:00 UTC. The circles on the flight paths mark the 119 AXBT drops. The 42 $\times$ 42 km^2 analysis box is also shown.

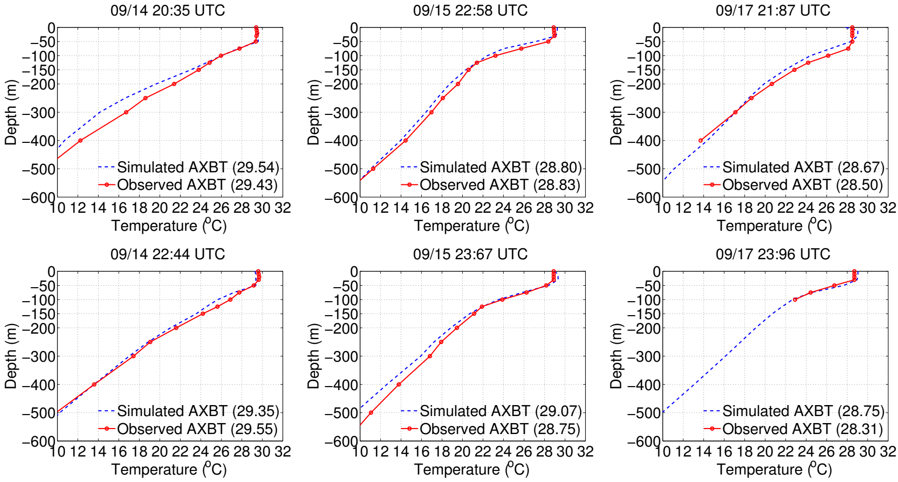
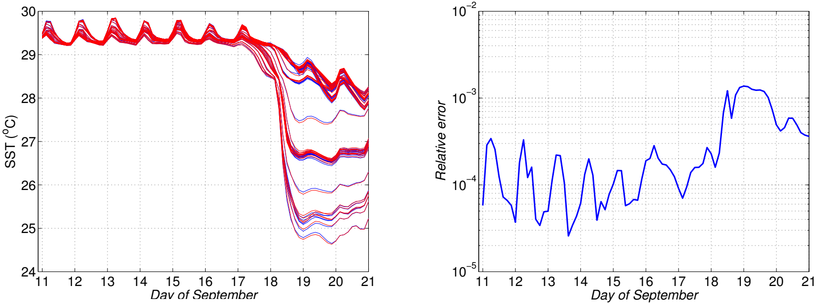


Figure: Comparison of HYCOM vertical temperature profiles with AXBT observations on Sep 14 (left), 15 (center) and 17 (right). Temperature averages over the first 50 m are shown in the legend.

PC Representation Errors



Evolution of the area-averaged SST realizations (blue) and of the corresponding PC estimates (red). The normalized rms error (right panel) remains below 0.1% for the duration of the simulation.

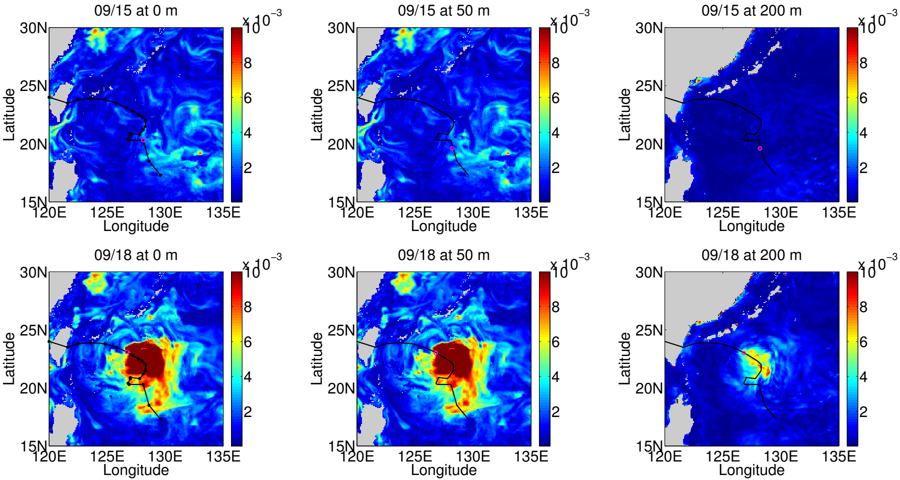
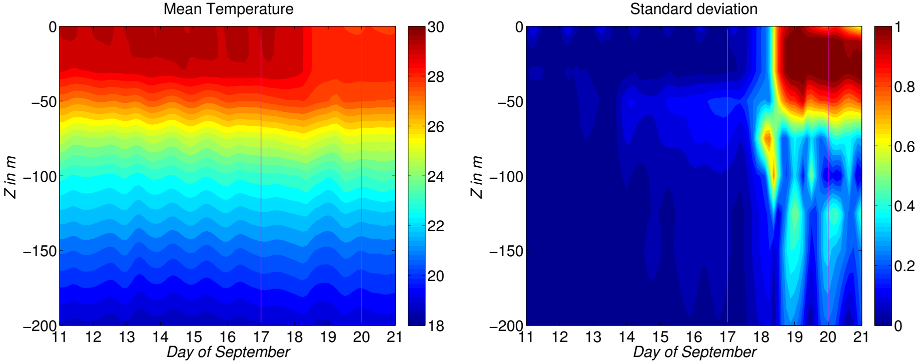


Figure: Normalized error between realizations and the corresponding PC surrogates at different depths; Top row: 00:00 UTC Sep 15; bottom row: 00:00 UTC Sep 18.

Depth Profile of Temperature Statistics



50m-deep mixed layer
2°C cooling after Fanapi arrives
Uncertainties confined to top 50 m.

SST Response Surface

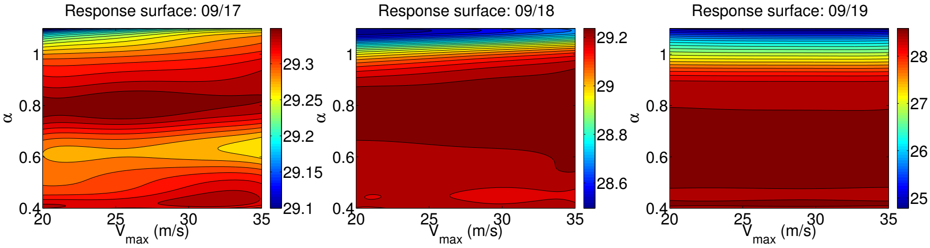


Figure: SST response surface as function of α and V_{max}, with fixed $m = 0$. Plots are generated on different days, as indicated. SST's dependence on $V_{\max}$ decreases after 09/17.

Markov Chain Monte Carlo

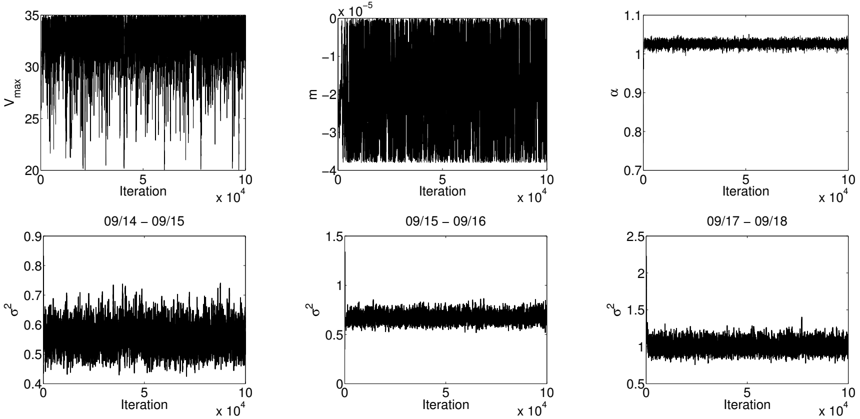


Figure: Top row: chain samples for V_{max}, m and α. Bottom row: chain samples for σ^2 generated for different days, as indicated.

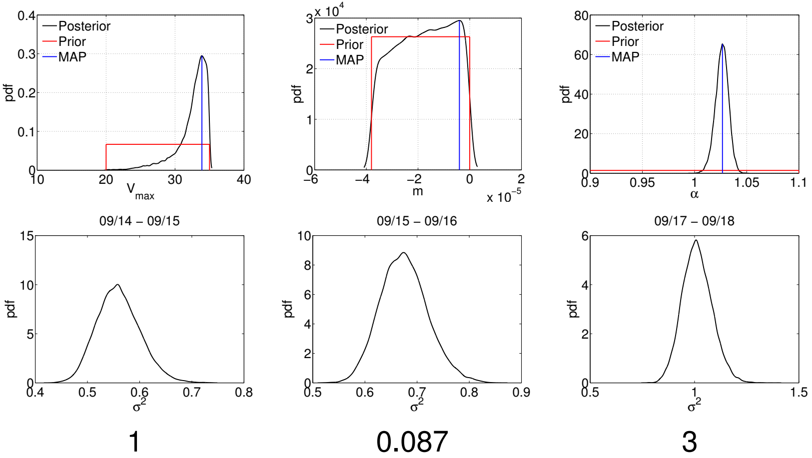


Figure: Posterior distributions for the drag parameters (top) and the variance between simulations and observations (bottom). The numbers show the Kullback-Liebler divergence quantifying the distance between 2 prior and posterior pdfs, i.e. the information gain.

Remarks on posteriors

- $V_{\max}$ exhibits a well-defined peak at 34 m/s.
- Posterior of m resembles prior. Data added little to our knowledge of m.
- α shows a definite peak at 1.03 with a Gaussian like-distribution.
- $\sqrt{\sigma^2}$ is a measure of the temperature error expected. This error grows with time from about 0.75° to 1°C.

Joint posterior PDFs

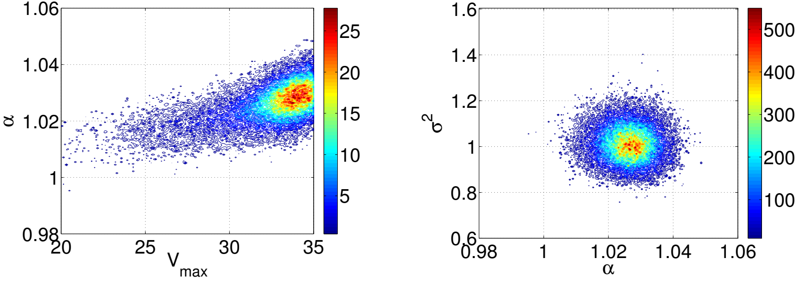


Figure: Left: joint posterior distribution of α (left) and V_{max}; right: joint posterior of α and σ^2, generated for Sep 17-Sep 18. Single peak located at $V_{\rm max} = 34$ m/s and $\alpha = 1.03$. The posterior shows a tight estimate for α with little spread around it.

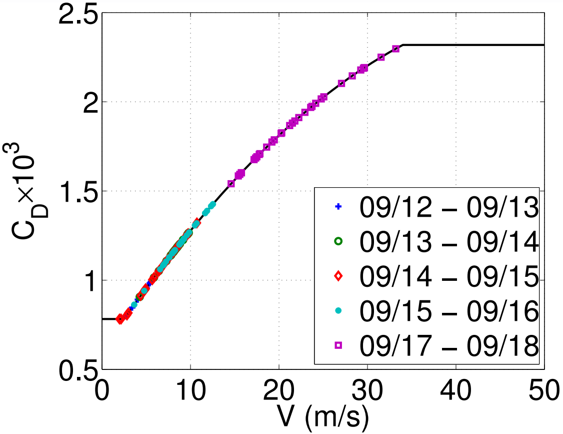


Figure: Optimal wind drag coefficient C_D using MAP estimate of the three drag parameters. The symbols refer to AXBT data used in the Bayesian inference.

Variational Form

- maximize the posterior density, or equivalently, minimize the negative of its logarithm

$$\mathcal{J}(\alpha, V_{max}, m, \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2) = \sum_{d=1}^{5} \left[J_d + \left(\frac{n_d}{2} + 1 \right) \ln(\sigma_d^2) \right] \ , \tag{7}$$

where J_d is the misfit cost for day d, the $\ln(\sigma_d^2)$ terms come from the normalization factors of the Gaussian likelihood functions and from the Jeffreys priors.

- The expression for J_d is:

$$J_d(\alpha, V_{max}, m, \sigma_d^2) = \frac{1}{2\sigma_d^2} \sum_{i \in \mathcal{I}_d} [M_i \ - \ T_i]^2 \ , \tag{8}$$

where $\mathcal{I}_d$ is the set of n_d indices of the observations from day d.

Adjoint-Free Gradients

Minimization requires cost function gradients

$$\left[\frac{\partial\mathcal{J}}{\partial\alpha},\frac{\partial\mathcal{J}}{\partial V_{\max}},\frac{\partial\mathcal{J}}{\partial m}\right] = \sum_{d=1}^{5}\frac{1}{\sigma_d^2}\left(\sum_{i\in\mathcal{I}_d}(M_i - T_i)\left[\frac{\partial M_i}{\partial\alpha},\frac{\partial M_i}{\partial V_{max}},\frac{\partial M_i}{\partial m}\right]\right)$$

Compute them from PC expansion

$$\left[\frac{\partial M}{\partial\alpha},\frac{\partial M}{\partial V_{\max}},\frac{\partial M}{\partial m}\right] = \sum_{k=0}^{P}\hat{M}_k(\boldsymbol{x},t)\left[\frac{\partial\psi_k}{\partial\alpha},\frac{\partial\psi_k}{\partial V_{\max}},\frac{\partial\psi_k}{\partial m}\right].$$

- $\frac{\partial\psi_k}{\partial\alpha}$ easy to compute
- No adjoint model needed
- For Hessian just differentiate above again.

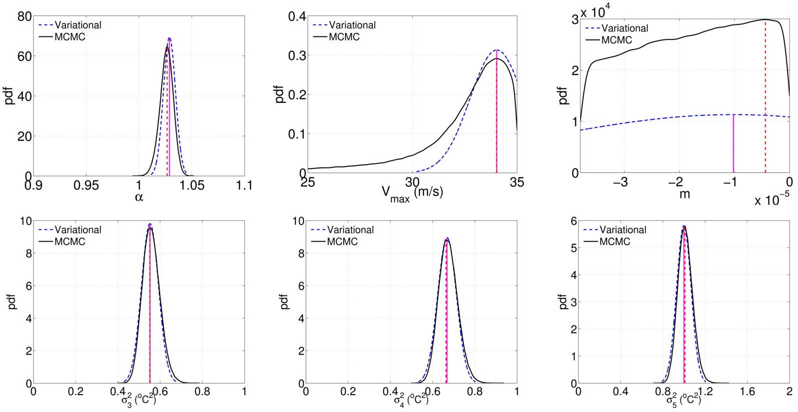


Figure: Posterior probability distributions for (top) drag parameters and (bottom) variances σ_d^2 at selected days using variational method and MCMC. The vertical lines correspond to the MAP values from MCMC and optimal parameters found using the variational method.

Conclusions & Future Work

- Identified drag parameters from ITOP observations during typhoon Fanapi.
- PC instrumental to make calculations tractable either through MCMC or through adjoint-free minimization
- $V_{\text{max}} \approx 34$ m/s
- Data uninformative regarding decrease in C_D
- C_D^{max} peaking around 2.3×10^{-3}
- Surface temperature measurements more valuable than ones at depths > 75 m.
- Inference of V_{max} and m hampered by lack of observation at wind speeds > 35 m/s.
- Future: Hurricane Model & other air-sea exchange coefficients

Publications

- I. Sraj, M. Iskandarani, A. Srinivasan, W. C. Thacker, and O.M. Knio, Computing Model Gradients from a Polynomial Chaos based Surrogate for an Inverse Modeling Problem *Monthly Weather Review*, in revision.
- J. Winokur, P. Conrad, I. Sraj, M. Iskandarani, A. Srinivasan, W.C. Thacker, Y. Marzouk, O. M. Knio, A priori testing of sparse adaptive polynomial Chaos expansions using an OGCM database, *Computational Geosciences*, in review.
- I. Sraj, M. Iskandarani, A. Srinivasan, W. C. Thacker, J. Winokur, A. Alexanderian, C-Y Lee S. S. Chen, O.M. Knio, Bayesian Inference of Drag Coefficient Parameters using AXBT data from Typhoon Fanapi, *Monthly Weather Review*, doi:10.1175/MWR-D-12-00228.1.
- A. Alexanderian, J. Winokur, I. Sraj, M. Iskandarani, A. Srinivasan, W. C. Thacker, and O. M. Knio, Global sensitivity analysis in an ocean general circulation model: a sparse spectral projection approach, *Computational Geosciences*, **16**, Vol 3, pp 757–778, 2012.
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Powell, M. D., P. J. Vickery, and T. A. Reinhold, 2003: Reduced drag coefficient for high wind speeds in tropical cyclones. *Nature*, **422**, 279–283, doi:doi:10.1038/nature01481, wind measurement and wind stress calculation in (observed) hurricane conditions.