Gent-McWilliams and isoneutral mixing parameterization in generalized vertical coordinates

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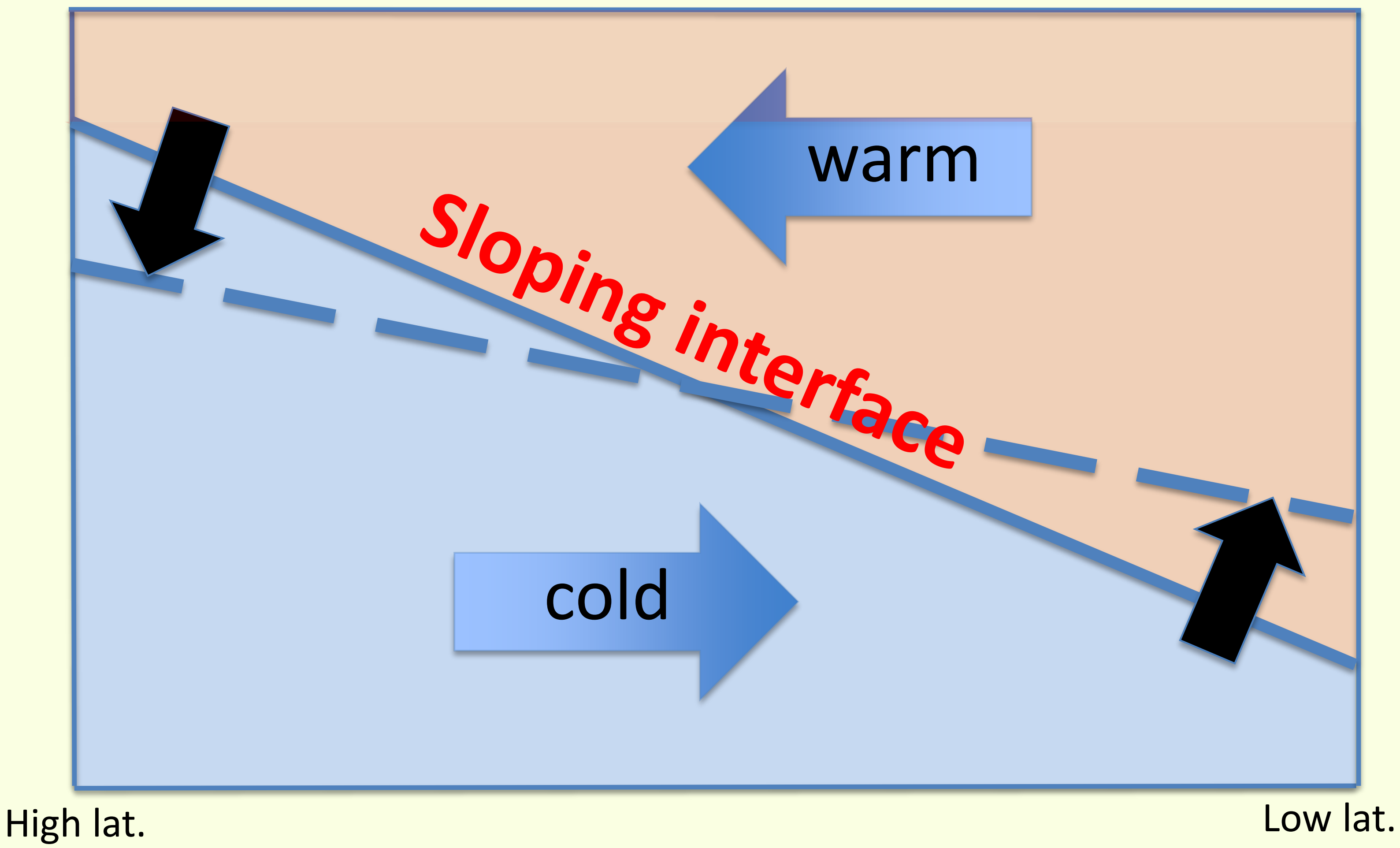
Baroclinic instability effects in a 2-layer ocean:



From Gent & McWilliams, 1990:

a. Eddy-resolving models

In isopycnal coordinates, the adiabatic density conservation and incompressible continuity equations are combined to give an equation for the thickness, $\partial h/\partial\rho$, where ρ is density and $h(x, y, \rho, t)$ is the physical height of a density surface. The equation is

$$\frac{\partial^2 h}{\partial t \partial \rho} + \boxed{\nabla_\rho \cdot \left(\frac{\partial h}{\partial \rho}\mathbf{u}\right)} = 0, \qquad (1)$$

If an eddy-resolving, adiabatic, isopycnal model is run to a statistical steady state, then the approximate balance in Eq. (1) will be

$$\boxed{\nabla_\rho \cdot \left(\frac{\partial \bar{h}}{\partial \rho}\bar{\mathbf{u}}\right) + \boxed{\overline{\nabla_\rho \cdot \left(\frac{\partial h'}{\partial \rho}\mathbf{u}'\right)}}} \approx 0. \qquad (5)$$

Here the thickness and velocity are divided into large-scale, time-mean $(\bar{\mathbf{u}})$ and eddy components $(\mathbf{u}')$, and the overbar represents an average over the eddy scales. The second term of Eq. (5) is the isopycnal mixing contribution due to eddies, and it appears as a source term in the thickness equation for the large-scale variables.

b. Non-eddy-resolving models

Therefore, the eddy mixing can be represented in approximate non-eddy-resolving models by the equation

$$\frac{\partial^2 h}{\partial t \partial \rho} + \nabla_\rho \cdot \left(\frac{\partial h}{\partial \rho}\mathbf{u}\right) + \boxed{\nabla_\rho \cdot \mathbf{F}} = 0. \qquad (6)$$

Part 1. GM parameterization

- If an ocean model does not resolve mesoscale eddies, baroclinic instability and its effect on the ocean state must be parameterized. This has become known as Gent-McWilliams (GM) parameterization.

- Baroclinic instability feeds on meridional buoyancy contrasts and has the effect of reducing them. In other words, it reduces the large-scale meridional slope of isopycnals in the fluid.

- GM captures this effect via a "bolus" mass flux which in *z*-coordinate models is expressed in terms of the 3-D buoyancy gradient. In isopycnic coordinate models, the bolus flux can be obtained directly by flattening coordinate surfaces – in effect smoothing them.

- The algorithm proposed here extends the isopycnal-flattening approach to **nonisopycnic** models (including the nonisopycnic subdomain in hybrid coordinate models).

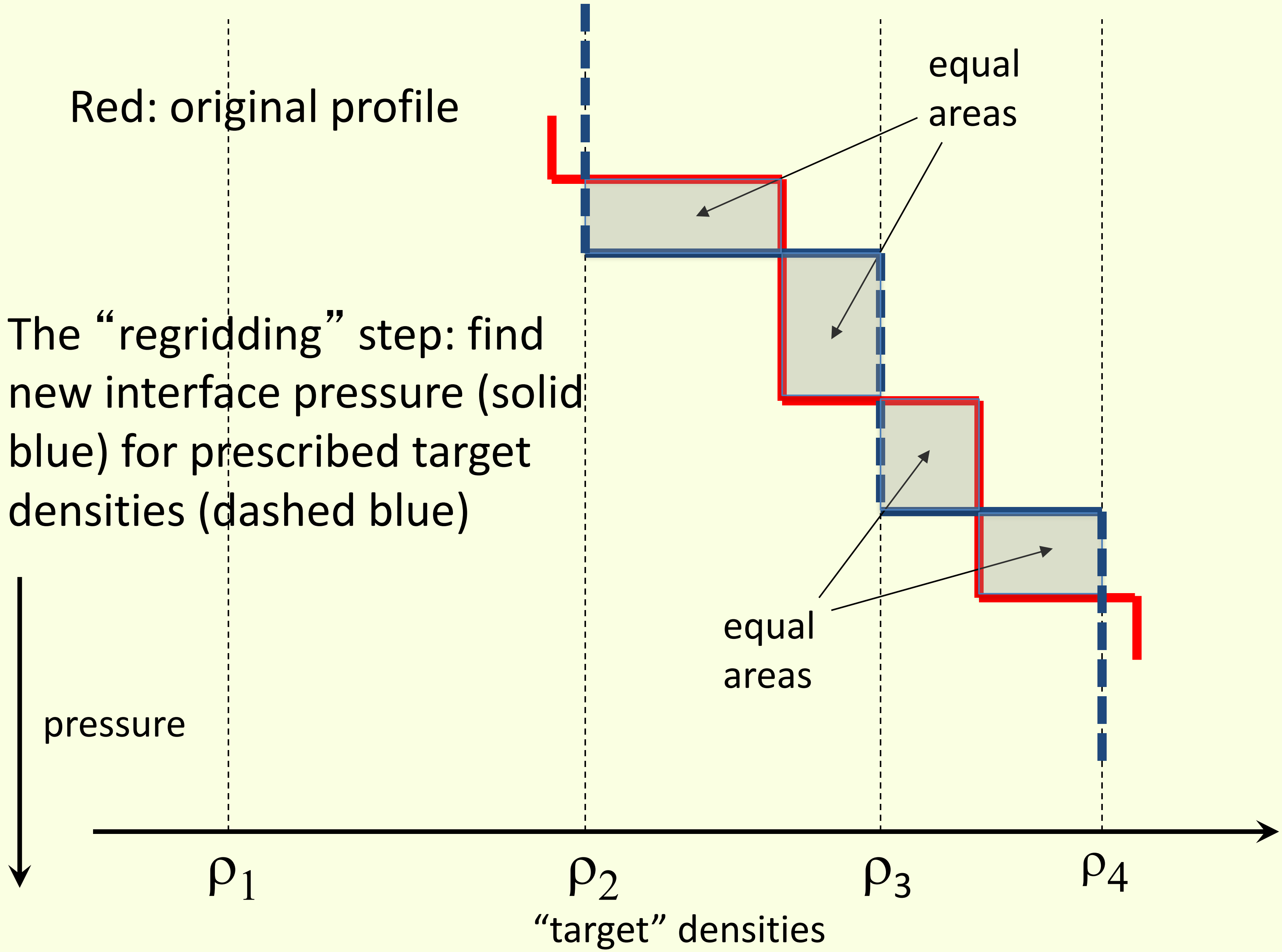
Approach:

1. Transform layer interface pressure from native to isopycnic grid
   - isopycnic subdomain in native grid remains unchanged
   - "Native" grid can be any grid, including *z, p,* or σ

2. Smooth interfaces and deduce corresponding bolus fluxes
   - By design, bolus fluxes have zero barotropic component

3. Transform bolus fluxes back to native model grid
   - Transform should conserve column integral of fluxes

4. Add bolus fluxes to flow field for transport of mass and tracers.



Red: original profile
The “regridding” step: find new interface pressure (solid blue) for prescribed target densities (dashed blue)
equal areas
equal areas
pressure
ρ_1
ρ_2
ρ_3
ρ_4
“target” densities

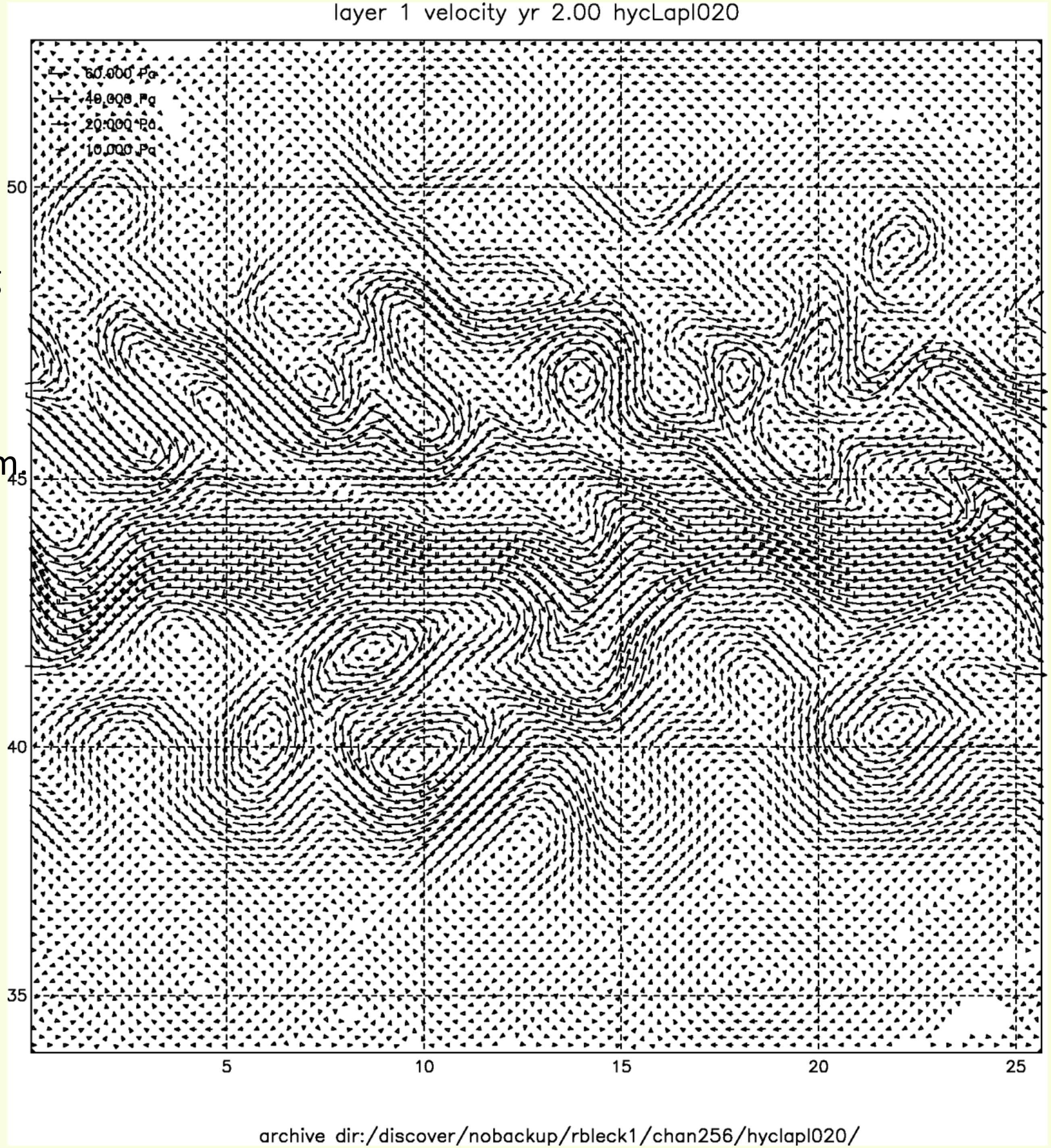
Remarks

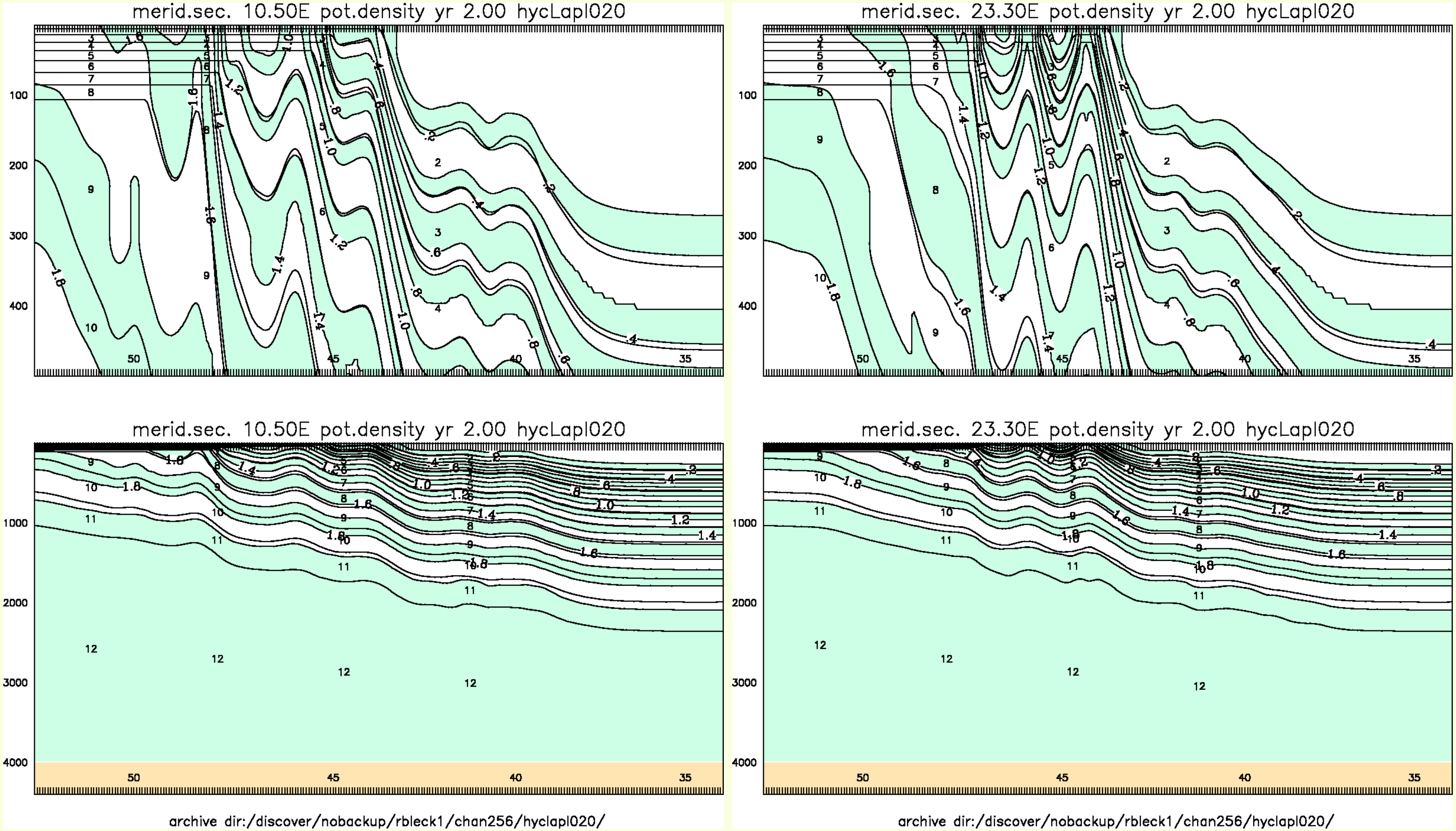
- Numerically robust even in weakly stratified regions (steeply inclined isopycnals)
- Not clear how to treat outcrop points where isopycnals often are near-vertical
  - Leave outcrop location fixed => **no** smoothing => no bolus flux at outcrop
  - Extend outcropped isopycnals along ocean surface => **strong** smoothing at outcrop => strong bolus flux re-stratifying mixed layer
  - Something in-between, possibly using guidance from *z*-coordinate community ("tapering" … see last slide)

Experiments

- Midlatitude baroclinic channel. 3 resolutions:
  - * 256 x 256 x 12 cells, mesh size 0.1 deg
  - * 64 x 64 x 12 cells, mesh size 0.4 deg
  - * 32 x 32 x 12 cells, mesh sie 0.8 deg
- 3 vertical coordinates: z, density, hybrid
- Thermal forcing (interface steepening) via Newtonian relaxation
- GM versus non-GM interface smoothing

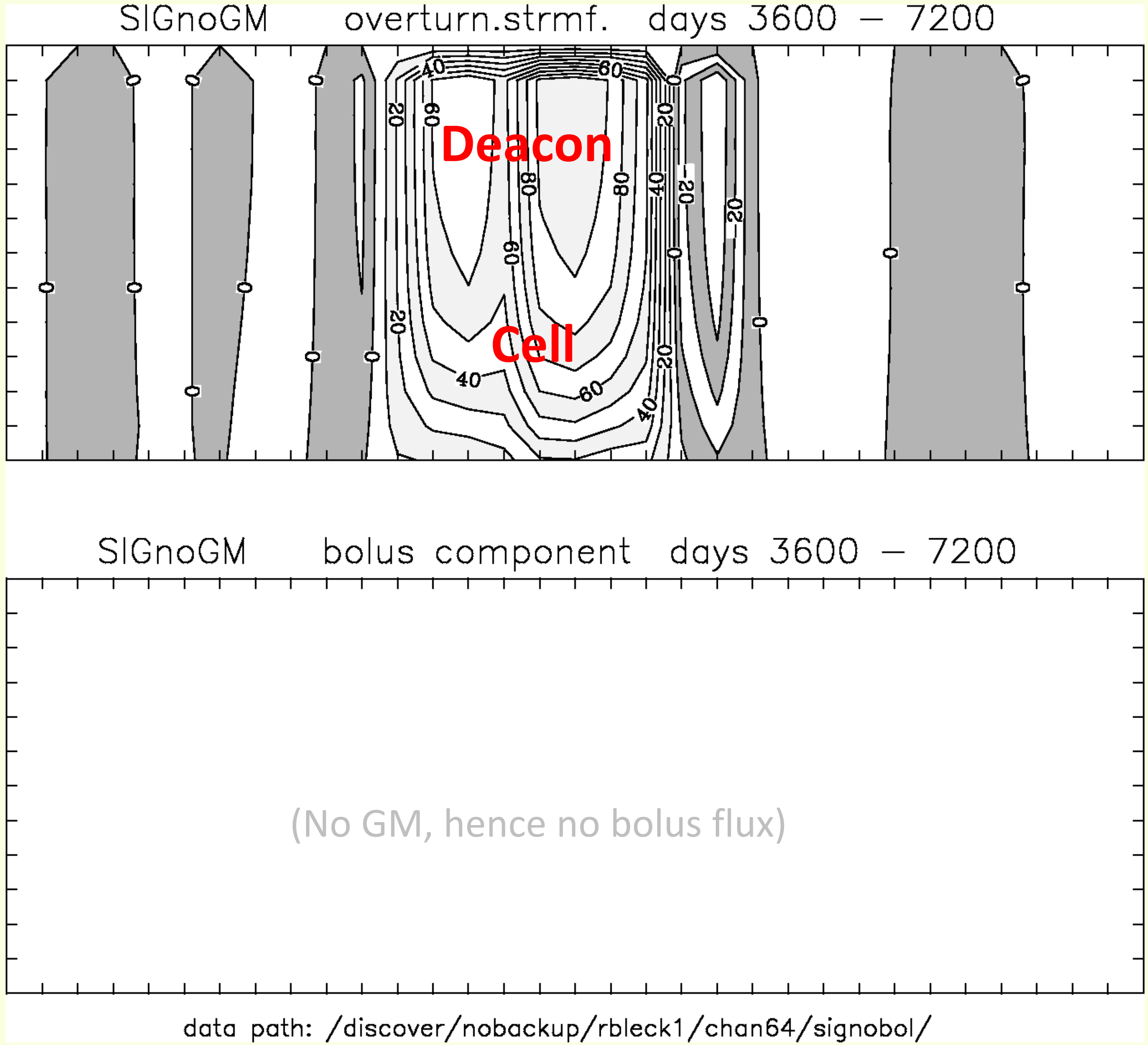
Sample snapshot of surface flow at 0.1 deg resolution. Mercator projection; latitude/longitude marked on side/bottom.



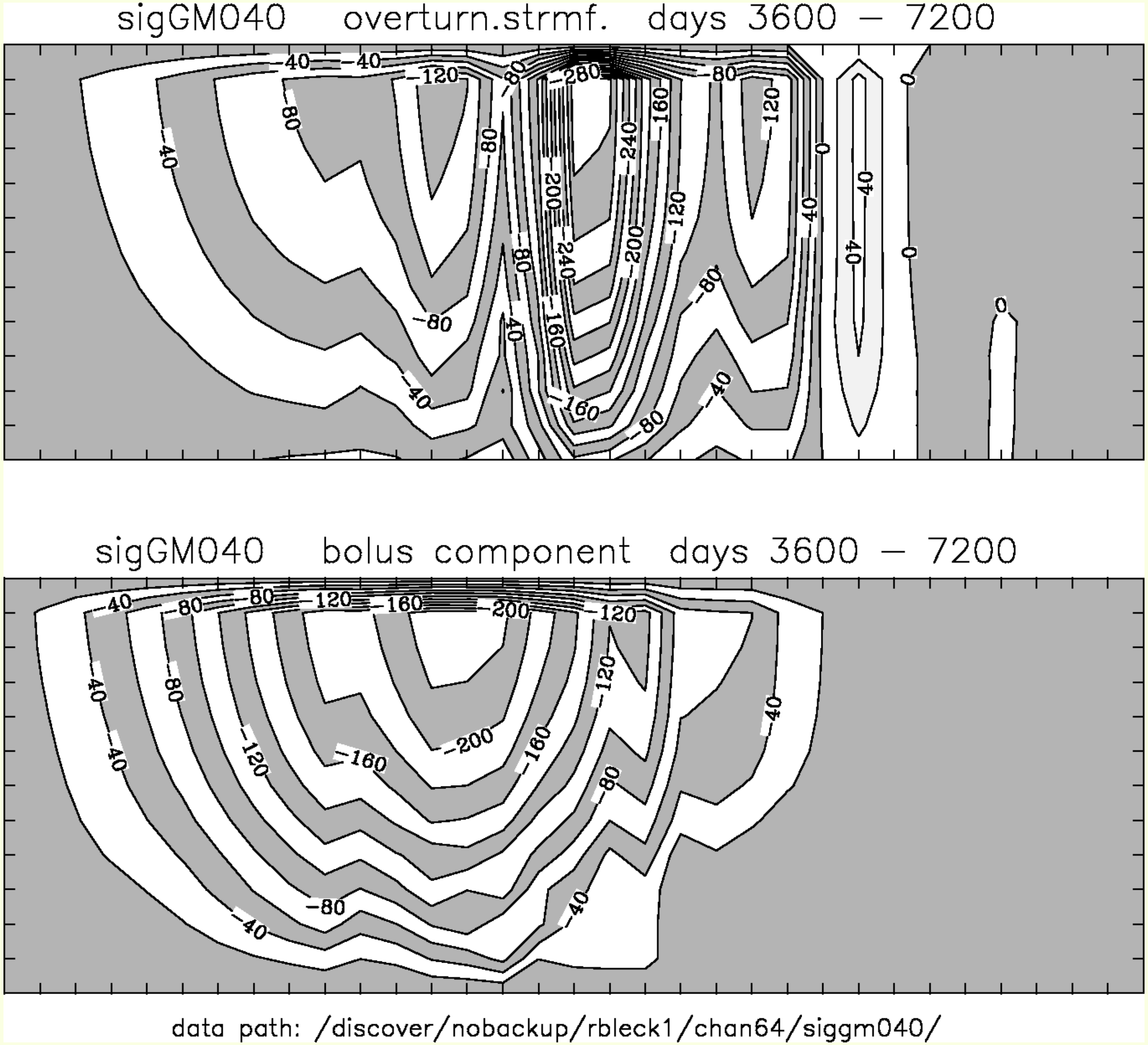


Sample meridional cross sections through HYCOM output field shown in previous slide, showing layer interfaces (solid) and density (shaded). Lower sections are full-depth; upper sections show details in top 500m. Latitude marked at bottom.

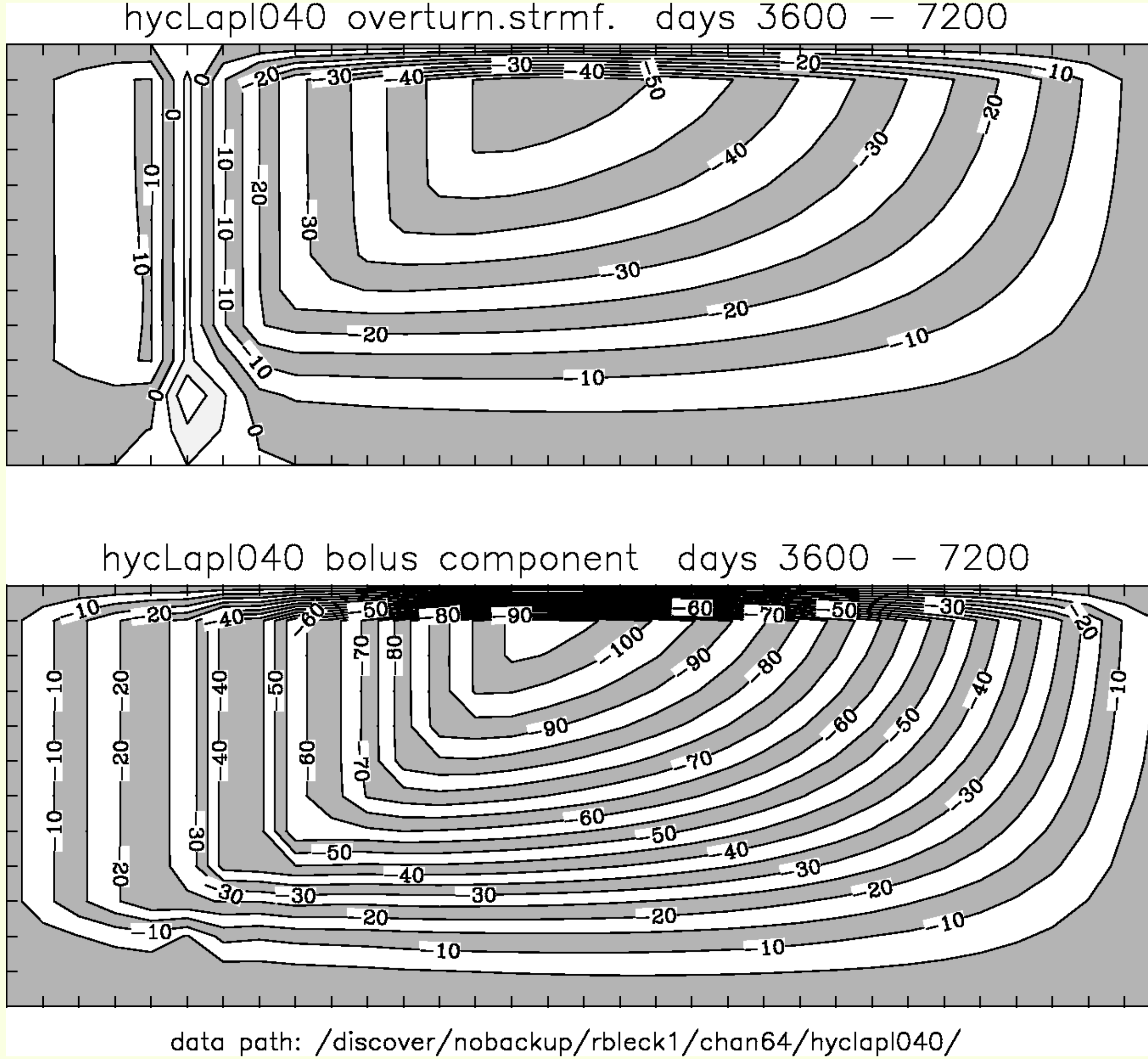
Overturning stream function (10-yr avg) at 0.4 deg resolution. Dark [light] shading indicates thermally direct [indirect] circulation. HYCOM configured as pure *p*-coord. model. **No GM**



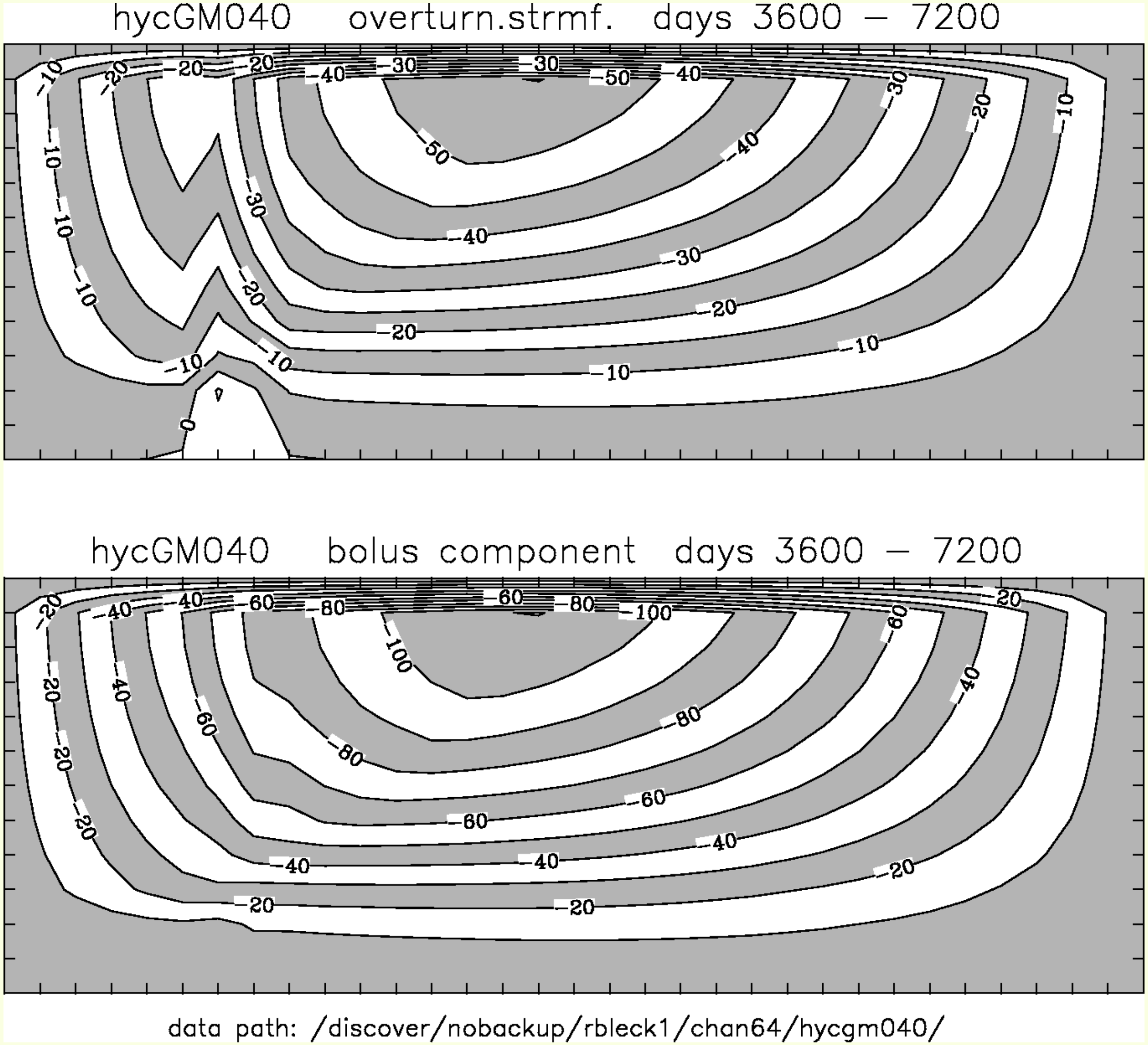
Overturning stream function (10-yr avg) at 0.4 deg resolution. Dark [light] shading indicates thermally direct [indirect] circulation. HYCOM configured as pure *p*-coord. Model. **GM activated**



Overturning stream function (10-yr avg) at 0.4 deg resolution. Dark [light] shading indicates thermally direct [indirect] circulation. Regular HYCOM coord. Configuration. **No GM** (but conventional interface smoothing)



Overturning stream function (10-yr avg) at 0.4 deg resolution. Dark [light] shading indicates thermally direct [indirect] circulation. Regular HYCOM coord. configuration, **GM activated**



Part 2. Isoneutral mixing ("Redi" mixing)

Approach:

- Solve tracer transport equation minus the lateral mixing term on native grid. Evaluate mixing term as follows.
- Step 1: Map transported tracer onto isopycnic grid (the grid constructed to obtain GM bolus fluxes)
- Step 2: Evaluate lateral mixing term

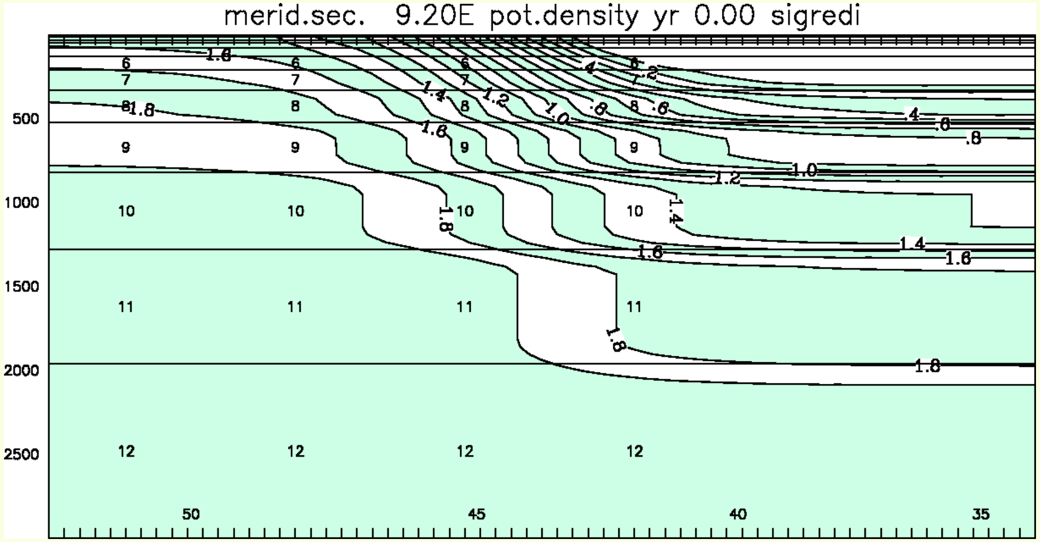
$$(\Delta p)^{-1}\nabla \bullet (K\Delta p\nabla T)$$

on isopycnic grid (K = eddy diffusivity, T = tracer, Δp = layer thickness, needed for conservation)

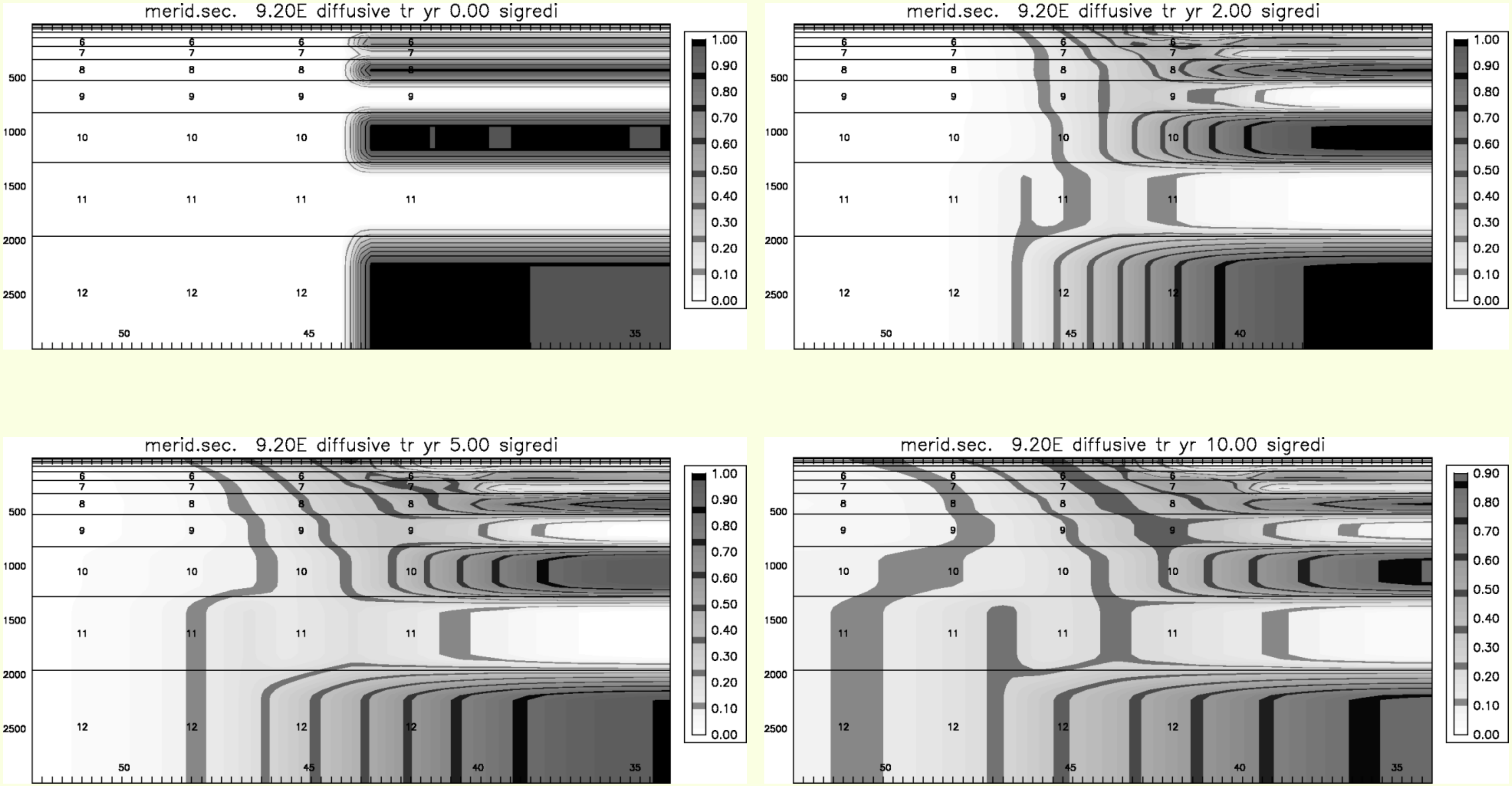
- Step 3: Map tracer increments back onto native grid

Isoneutral mixing (2)

- Tracer tendencies mapped back to native grid may violate positive-definiteness and monotonicity of tracer on native grid
- Step 4: For this reason, add an extra vertical diffusion step on the native grid
- The variable diffused is the tracer amount exceeding some prescribed limit (for example, zero in case of a positive-definite tracer). No outlyers => no diffusion.
- This does not completely remove outlyers but dilutes them, thereby suppressing them over time.



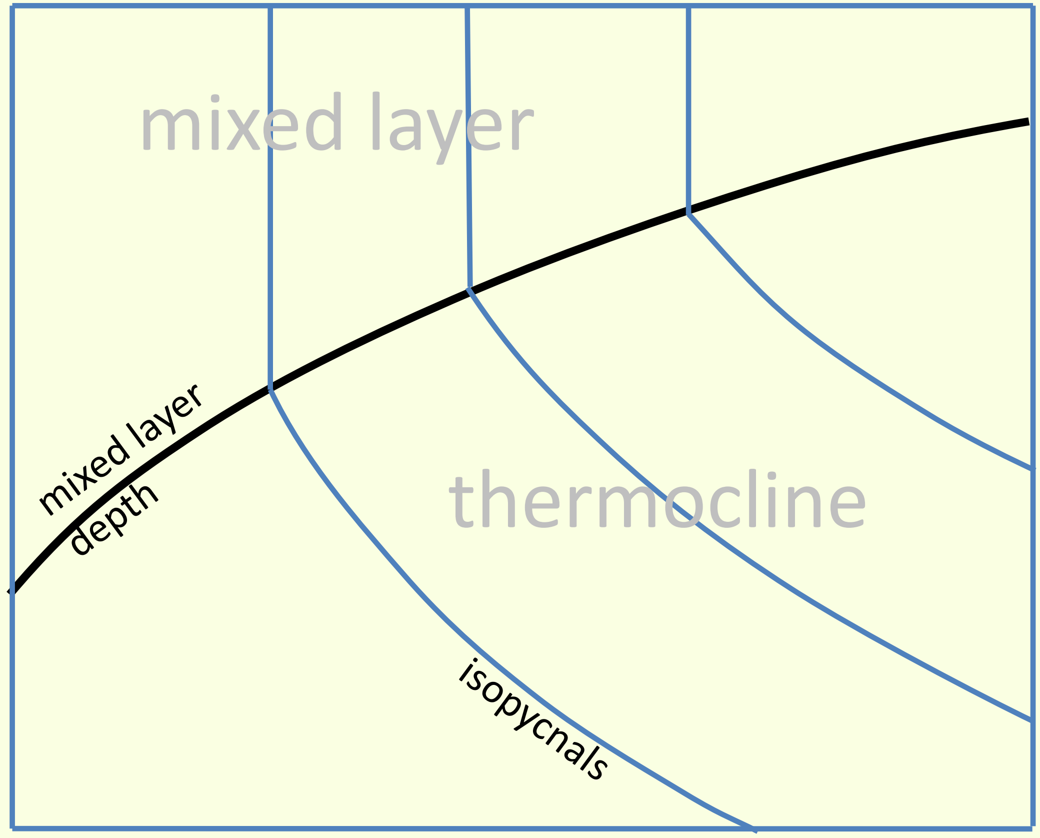
Vertical-meridional section through initial conditions on 64 x 64 grid, showing layer interfaces (horizontal lines) and density (shaded). HYCOM configured as pure *p*-coordinate model. Numbers 6 … 12 are layer indices. Latitude marked along bottom; depth marked at left. Stairstep appearance of isopycnals is intentional (density is a layer variable!). Tracer shown in next frame is being diffused along these isopycnals.



Vertical sections showing passive tracer at initial time (top left) and after being diffused for 2,5,10 years. Density field is shown in previous slide. HYCOM configured as pure *p*-coordinate model.

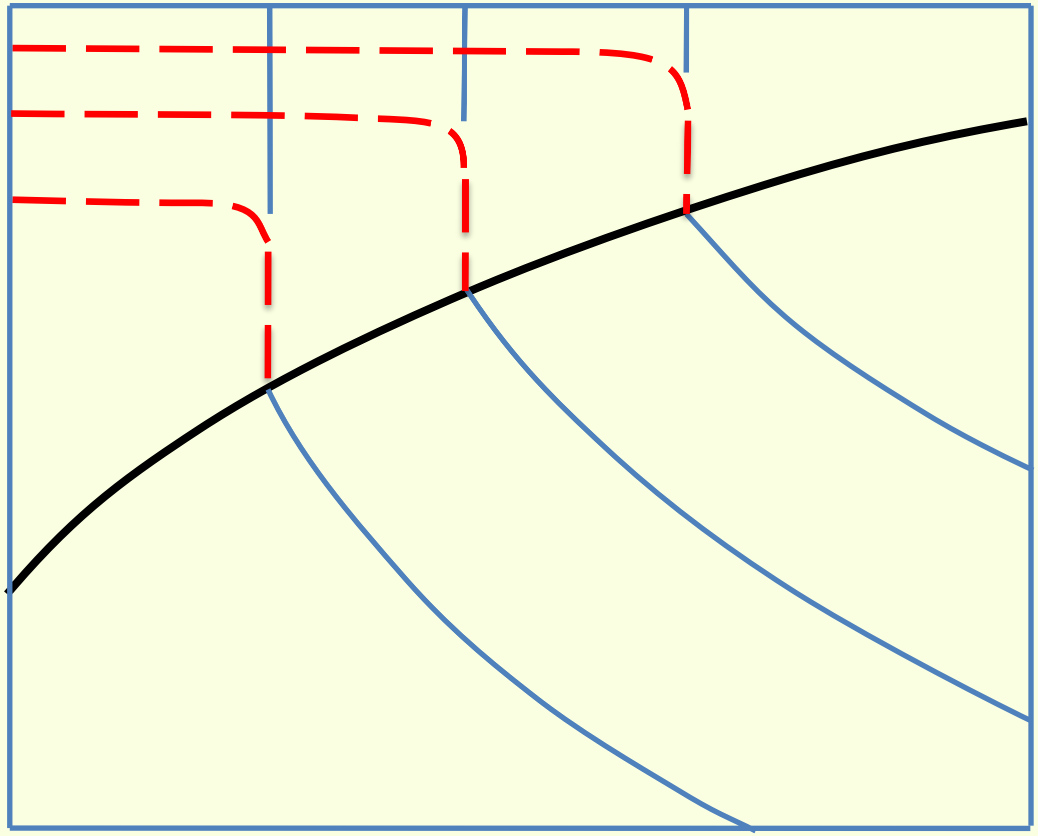
Remarks

- GM and tracer fluxes have been computed on a purely isopycnic grid.
- Mixed-layer "tapering" can be emulated by gradually reducing the slope of isopycnals as they rise to the surface (see last slide).

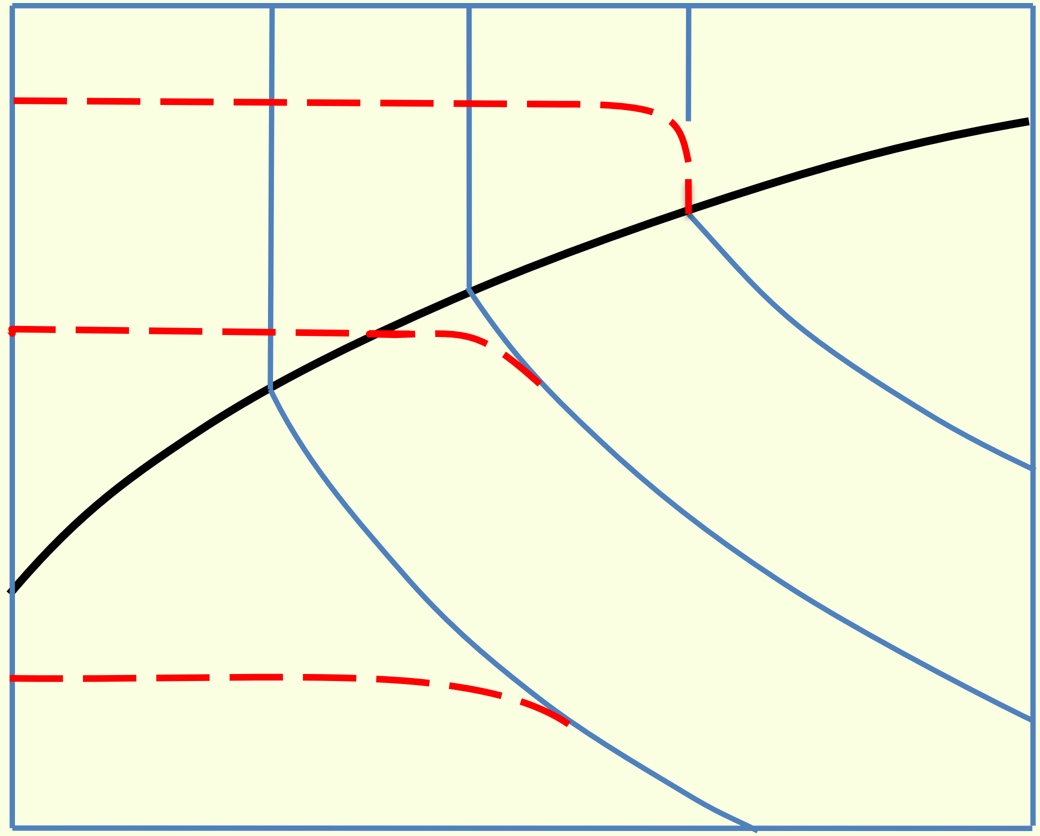


Idealized vertical-meridional section showing mixed layer and several isopycnic layers. Surface density increases to the left.

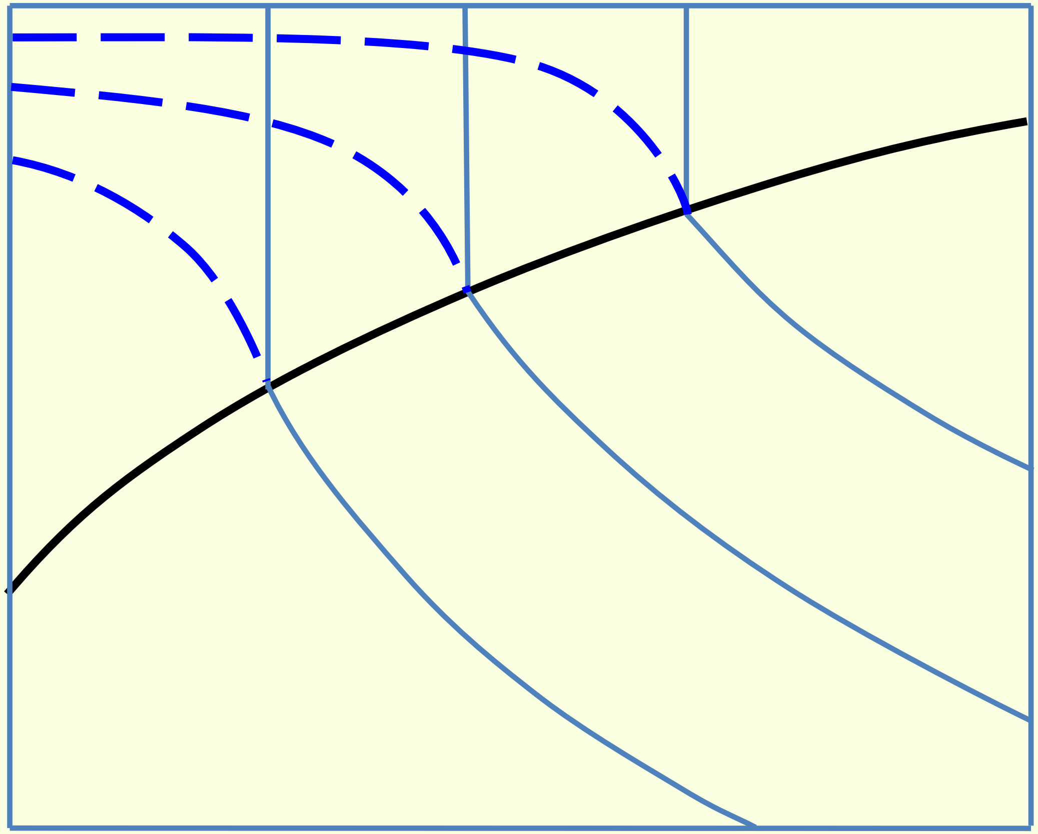
Interface placement in HYCOM (schematic). **Small** min. lyr.thicknss



Interface placement in HYCOM (schematic). **Large** min. lyr.thicknss



Suggested interface placement for near-surface "tapering" of GM and Redi fluxes



Assessment:

- Works in mid-latitude channel
- However, isopycnal slopes in Southern Ocean will require extra steps to assure numerical stability

New idea for HYCOM (credit: Eric Chassignet)

- Run interface smoothing <u>and</u> conventional GM parameterization (based on horizontal density gradient) simultaneously.
- One process will be active in HYCOM's isobaric subdomain, the other one in the isopycnic subdomain => **no duplication!**